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The conceptualization of mathematics teachers' professional competence in project-based learning contexts

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Abstract

The impact of teacher competence on teaching practice has been increasingly prominent, especially in mathematics education, but little research focuses on teacher competence in project-based learning (PjBL) contexts. Drawing on existing models and frameworks, mathematics teachers' professional competence in PjBL contexts was conceptualized as a multi-dimensional construct with cognitive facets (mathematics pedagogical content knowledge related to PjBL, namely P-MPCK) and affective-motivational facets (teacher beliefs and teacher self-efficacy). The conceptual model was empirically validated through an assessment instrument combining cognitive tests (e.g., situated text-vignette tasks) and affective scales. Results suggest that teacher beliefs and self-efficacy were loaded on three factors respectively and the four-dimension model of P-MPCK had strong goodness of fit, which were consistent with the conceptual model. This study provides valuable insights for preparing mathematics teachers to teach PjBL and supporting their professional development.

Keywords: teacher competence, project-based learning, assessment instrument, mathematics teacher education

INTRODUCTION

Project-based learning (PjBL), an educational approach rooted in constructivist theories of learning (Ravitz, 2010), has gained increasing recognition worldwide. With the national emphasis on curriculum reform of basic education, PjBL, has been supported and advocated at the policy level in China. Mathematics curriculum standards for compulsory schools incorporate PjBL to foster students' problem-solving abilities and deepen their understanding of mathematics (Ministry of Education, 2022), because this approach allows students to engage with mathematical concepts profoundly and meaningfully, which is often absent in conventional teaching methods. The integration of PjBL in mathematics education is not merely a trend, but a strategic move to enhance students' mathematical learning experiences (Han et al., 2016) and motivation to learn mathematics (Holmes & Hwang, 2016).

Nevertheless, the transition to PjBL involves several difficulties. Meyer et al. (1997) and Aldabbus (2018) underscored the challenges of PjBL in an actual mathematics classroom, including the choice of content, time management, project assessment, and resource inadequacy. To address these dilemmas effectively, preparing mathematics teachers for PjBL is critical, and mathematics teachers' competence has been considered as a crucial factor associated with instructional quality and students' mathematics achievement (Yang & Kaiser, 2022). Thus, it is necessary to clarify which specific competence mathematics teachers should have in PjBL contexts. While the professional competence of mathematics teachers has been investigated and measured via cognitive or situational approaches (Blömeke et al., 2020; Kaiser et al., 2017), little research has a focus on mathematics teachers' competence in PjBL contexts. However, drawing on prior research on teacher competence and PjBL, it is likely to conceptualize mathematics teachers' competence in PjBL context. Teacher-training programs on PjBL have been widely conducted to provide an empirical foundation for validating the theoretical model and measuring mathematics teachers' competence in PjBL contexts. To sum up, we suggest that an understanding of mathematics teachers' competence in PjBL contexts will

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Contribution to the literature

- Proposes a novel conceptual model integrating mathematics pedagogical content knowledge related to PjBL (P-MPCK), teacher beliefs, and teacher self-efficacy, addressing the underexplored multi-dimensional competence of mathematics teachers in PjBL contexts, particularly in non-Western educational settings.
- Develops a validated assessment instrument combining affective scales and situated text-vignette tasks, enhancing measurement of teachers' diagnostic and intervention skills in PjBL.
- Demonstrates significant correlations between teachers' P-MPCK, constructivist beliefs (CBF), and self-efficacy, offering evidence-based insights for teacher education and PjBL implementation strategies.

provide valuable insights into preparing teachers to teach PjBL and supporting their professional development.

LITERATURE REVIEW

Conceptualization of Teacher Competence

The concept of *competence* in educational research was derived from Weinert (2001), who describes competence as a multi-dimensional construct comprising of cognitive and affective-motivational facets. On the one hand, the cognitive facets of teacher competence refer to teachers' professional knowledge, which according to Shulman's (1986) work can be distinguished into content knowledge (CK, namely domain-specific subject-matter knowledge), pedagogical content knowledge (PCK, i.e., knowledge of instructional strategies and representations, and knowledge of students' (mis)conceptions), and general pedagogical knowledge (GPK, including broad principles and strategies of classroom management and organization). In mathematics education, teachers' professional knowledge mainly consists of mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK), although it has been conceptualized in various ways (Neubrand, 2018). For example, Shulman's (1987) conceptualization of teachers' knowledge with seven categories was refined and integrated into within one overarching category of knowledge, the mathematics knowledge for teaching, only covering categories related to CK and PCK (Ball et al., 2008). Moreover, it is difficult to theoretically and empirically make a distinction between MCK and MPCK so that less consensus about the components of MPCK exists (Depaepe et al., 2013).

On the other hand, the affective-motivational facets of teacher competence include beliefs and affective traits (e.g., motivation), as well as metacognitive abilities (e.g., self-regulation), which build the bridge between cognitive abilities and teaching practice (Döhrmann et al., 2012). The conceptualization of beliefs is as complex as that of knowledge, given that they are shaped by a variety of factors and are subject to individual interpretation and social construction (Richardson, 1996). Teachers' belief systems are multifaceted, encompassing views on students' abilities, the purpose of mathematics education, and the teacher's role (Thompson, 1992). Hoy et al. (2006) identified three critical aspects of teacher beliefs:

(a) beliefs about the role of teachers or their abilities, namely teachers' *self-efficacy* (Tschannen-Moran et al., 1998),

(b) *epistemological beliefs* (Hofer & Pintrich, 1997; e.g., beliefs about the nature of mathematical knowledge and the knowing of mathematics), and

(c) *beliefs about teaching and learning* (in mathematics education as beliefs about mathematics teaching and learning; Liljedahl et al. 2021).

From the perspective of philosophy, Ernest (1989) classified epistemological beliefs about mathematics into three views: "the instrumentalist view of mathematics as a set of unrelated but utilitarian rules and facts … the Platonist view of mathematics as a static but unified body of certain knowledge … and the problem-solving view of mathematics as a dynamic, continually expanding field of human creation and invention, a cultural product". From the perspective of learning theories, both epistemological beliefs and beliefs about teaching and learning can be classified into *transmissive-oriented beliefs* where teachers tend to convey knowledge to students, and *constructivist-oriented beliefs* where teachers prefer to apply a learner-oriented strategy (Voss et al., 2013). The instrumentalist and Platonist views correspond to transmissive-oriented beliefs whereas the problem-solving view corresponds to constructivist-oriented beliefs (Beswick, 2005).

Defined in this way, teacher competence is regarded as the sum of personal dispositions (cognition and affect-motivation) associated with latent abilities or skills required for teaching performance (Klieme et al., 2008). To understand the level and depth of teacher competence and its impact on teaching performance, many scholars and research institutes have developed the conceptual frameworks of teacher competence, which were validated in large-scale empirical studies. For instance, in an international comparative *teacher education and development study–learning to teach mathematics* (TEDS-M), and the *cognitively activating instruction and the development of students' mathematical literacy* (COACTIV) research program, mathematics teachers' competence was conceptualized as the integration of professional knowledge, beliefs, motivation, and self-regulation (Döhrmann et al., 2012; Kunter et al., 2013; Tatto et al., 2012). And these empirical studies on teacher professionalism were predominantly drawn on a cognitive perspective focusing on knowledge facets, which has been proved in other small-scale research on professional competence or knowledge of pre- and in-service teachers (Blömeke & Delaney, 2012).

Furthermore, the cognitive and affective-motivational resources may be not necessarily transformed into teaching performance in practice, although they underlie performance. Consequently, teacher competence as a continuum instead of a dichotomous characteristic was proposed by Blömeke et al. (2015), which highlights a situated perspective; that is, the interplay between teachers' dispositions and teaching performance are mediated by situation-specific skills in terms of perception, interpretation, and decision-making (i.e., noticing skills).

Measurement in Mathematics Teachers' Competence

Based on the two perspectives on teacher competence, cognitive or situated approaches were used to evaluate teachers' professional knowledge and skills (Kaiser et al., 2017). From a cognitive perspective, paper-and-pencil tests instruments for knowledge facets, such as MCK, MPCK, and GPK, have been widely applied in the field of mathematics educational research (Hill et al., 2004; Krauss et al., 2008). Some researchers developed particular instruments with corresponding factors to evaluate teachers' MCK on different topics, such as ratio and proportion (Ekawati et al., 2015), and space and shape (Ekawati, 2022). In the context of TEDS-M, König et al. (2011) designed a standardized test of GPK based on a matrix with teaching quality and cognitive demands. Conversely, the situated perspective considers the practical application of teachers' knowledge in real-world teaching scenarios. As for PCK, situated approaches, such as interviews, classroom observations, and mentoring meetings, have been used to capture the dynamic nature of teaching in some small-scale studies (Depaepe et al., 2013). Importantly, the majority of studies on MPCK have aimed to investigate the extent to which teachers understand multiple solutions of mathematical tasks, mathematics-specific instructional strategies, and students' misconception and difficulties (Krauss et al., 2008; Lo, 2020; Zhou et al., 2006).

Situation-specific skills may be classified as cognitive facets of the cognitive processes of classroom teaching rather than actual observable performance (Star & Strickland, 2008). As a result, mathematics-related and general pedagogical skills in three situated facets of noticing (perception, interpretation, and decision-making) have been assessed based on video vignettes (Copur-Gencturk & Tolar, 2022; König et al., 2015). König et al. (2014) found that mathematics teachers' ability to interpret correlated with GPK, but in turn GPK could not predict noticing skills in the future. To explore teacher competence in more situated and performance-oriented ways, a follow-up study of TEDS-M (TEDS-FU) has incorporated the concept of *noticing* into the original theoretical framework (Kaiser et al., 2015). The TEDS-FU results reveal that it is possible to integrate cognitive and situated approaches for a holistic understanding of teacher competence, because the cognitive approach provides a foundation for what teachers need to know for teaching, whereas the situated approach examines how teachers apply their knowledge and skills in practice (Kaiser et al., 2017). Nevertheless, challenges in developing assessment instruments of teacher knowledge remain, such as item formats and scoring scheme in video-based tests (Kaiser et al., 2015), or cross-cultural discrepancies in adaptation of instruments (Hsieh, 2013; Yang et al., 2018).

With respect to affective-motivational facets of teacher competence, researchers have preferred to scrutinize teachers' beliefs about the nature of mathematics (i.e., epistemological beliefs) and about mathematics teaching and learning through quantitative and/or qualitative approaches (Francis et al., 2014). From the methodological perspective, teacher beliefs could be surveyed through standardized self-reported instruments, including questionnaires, in which beliefs may be viewed as stable cognitive constructs detached from contextual factors (Hofer, 2004), but in order to address the decontextualized problem on beliefs, some researchers have introduced other approaches, such as case study via in-depth interviews (e.g., Beswick, 2012; Löfström & Pursiainen, 2015; Xenofontos, 2018). From the theoretical perspective, Ernest's (1989) three views of mathematics (instrumentalist, Platonist, and problem-solving) have been widely used in studies on teachers' mathematics beliefs. For example, Pagiling et al. (2021) found that preservice teachers had higher Platonist views in classes dominated by high ability students whereas they tended to have instrumentalist views in classes dominated by low ability students, and as for the nature of mathematics, the instrumentalist view was dominated. It is common that teachers have mixed beliefs in mathematics but with a lack of problem-solving views (Beswick, 2005).

To further understand teachers' problem-solving views, researchers have started to focus on constructivist-oriented beliefs in the context of educational reform. For example, Cormas (2022) found that pre-service mathematics teachers' beliefs became more constructivist-based after implementing problem-solving lessons. Similarly, Russo et al. (2020) found that most teachers held positive beliefs about struggle (i.e., challenging problem solving), but pointed out the level of challenge should be suitable for students. Apart from beliefs related to mathematics, Saadati et al. (2019) also explored mathematics teachers' self-efficacy in teaching and doing complex problem-solving tasks. By contrast, Li et al. (2024) centered on mathematics teachers' self-efficacy in cooperative learning.

Teacher Competence in PjBL Contexts

In western countries, numerous studies on teacher competence have been theoretically and empirically implemented but less is known about teacher competence in PjBL contexts. By contrast, researchers in China have been devoted to exploring teacher competence in PjBL contexts, because teacher competence can provide a crucial safeguard for instructional quality and student achievement (Yang & Kaiser, 2022), and some particular competence is needed for teachers to meet demands and challenges for PjBL in practice. For this reason, Zhang and Yang (2021) proposed a theoretical framework including seven teaching competencies in PjBL contexts from the perspective of designing and implementing the national curriculum. Building on a holistic model of competence (Le Deist & Winterton, 2005), a conceptual model and an assessment scale of teacher competence with three facets (cognitive competence, professional competence, and social competence) were developed by Wan and Gao (2023). Similarly, based on role analysis and needs assessment, Ye et al. (2021) determined four components of teacher competence using empirical results from grounded theory, including learning competence, design competence, collaborative problem-solving competence, and assessment and feedback competence.

As reviewed above, affective-motivational facets and subject-related characteristics (such as mathematics) were lacking in these frameworks, which were constructed to meet the national standards or requirements for the teacher role in PjBL contexts, rather than for examining the outcomes of teacher education program on PjBL–one of our goals in the present study. To understand mathematics teachers' competence in PjBL contexts, the following two main research questions (**RQ**s) were discussed:

RQ1. How to conceptualize mathematics teachers' competence in PjBL contexts?

RQ2. How to measure mathematics teachers' competence in PjBL contexts?

THEORETICAL FRAMEWORK

A salient feature of PjBL is to immerse students in collaborative and inquiry-based learning to create artifacts or look for solutions to real-world problems (Kokotsaki et al., 2016). Given the holistic construct of teacher competence, the outcomes of teacher education and the features related to PjBL, a theoretical framework of teacher competence for teaching mathematical modeling proposed by Wess et al. (2021) was chosen as a proper reference for our study. The first reason is that mathematics modeling is an ambitious teaching practice to teach students how to apply mathematics to solve authentic problems (Suh et al., 2018), which concurs with the aim of enhancing students' problem-solving skills through PjBL in math (Rehman et al., 2024), although mathematical modeling focuses on the creation of mathematical models for a real-world scenario whereas PjBL highlights the construction of real models using mathematics knowledge and methods. Second, the framework of teacher competence for teaching mathematical modeling (hereafter 'modeling framework') was drawn on the COACTIV model of mathematics teachers' professional competence (Kunter et al., 2013), including cognitive and affective-motivational facets. Third, based on the modeling framework, a test instrument (hereafter 'modeling instrument') was developed to examine the outcomes of teacher education aimed at teaching mathematical modeling. The modeling instrument has been scientifically designed and empirically tested, and its feasibility for localized application in China has been confirmed (Chen, 2023), so the modeling instrument could be theoretically adapted for investigating the teaching competence of PjBL in Chinese contexts. Last but not least, both the modeling framework and instrument center on modeling-specific PCK instead of routine MPCK (competence in the narrow sense: knowledge and skills), which is similar to PjBL-specific PCK emphasized in our teacher education programs.

Derived from the modeling framework, conceptual model of math teachers' professional competence in PjBL contexts (**Figure 1**) is presented in three facets: teacher beliefs, teacher self-efficacy, and P-MPCK.

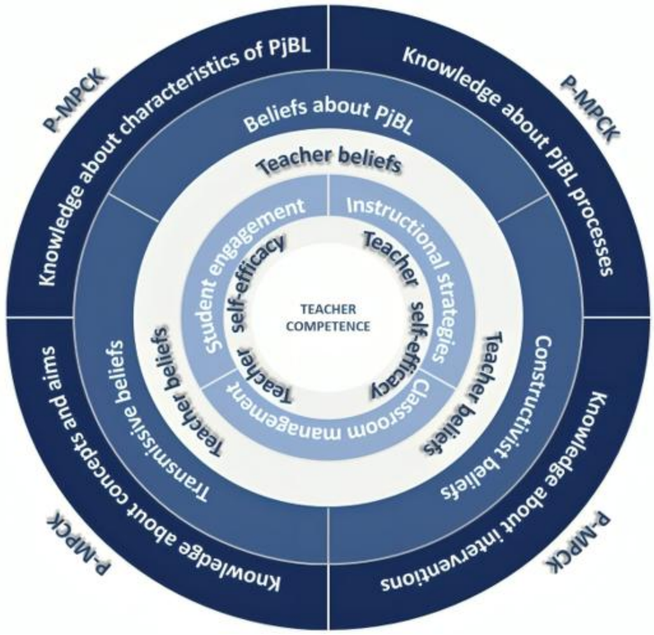

Figure 1. The conceptual model of mathematics teachers' competence in PjBL contexts (Source: Authors' own elaboration)

Teacher beliefs consist of three dimensions: beliefs about PjBL (BFP), constructivist beliefs (CBF), and transmissive beliefs (TBF). *BFP* refer to the extent to which teachers agree with the practical values of PjBL to students' achievement and development (Condliffe et al., 2017), and the goal/value of PjBL in mathematics education (Yunita et al., 2021). *CBF* measure the extent to which teachers agree with a learner-oriented strategy (Voss et al., 2013). Instead, *TBF* emphasize "the skill mastery with correct performance and conceptual understanding with unified knowledge" (Ernest, 1989) and measure the extent to which teachers agree with a teacher-oriented strategy (Voss et al., 2013).

Teacher self-efficacy is divided into three dimensions: self-efficacy for instructional strategies (SEI), self-efficacy for classroom management (SEC), and self-efficacy for student engagement (SES). They were derived from self-efficacy proposed by Tschannen-Moran and Hoy (2001) instead of those in the modeling framework. On the one hand, Wess et al. (2021) conceptualized teacher self-efficacy for diagnosing performance potentials for the activities of modeling and other mathematical working, which did not concern with the characteristics of the fusion of PjBL tasks and mathematics learning. On the other hand, three dimensions of teacher self-efficacy are associated with challenges in implementing PjBL in math; that is, teachers' self-efficacy depends on how they view the teaching difficulties in PjBL contexts. *SEI* represents the extent to which mathematics teachers believe themselves in implementing PjBL through appropriate instructional strategies, such as proposing good questions to students, and adjusting lessons suitable for the level of students. *SEC* represents the extent to which mathematics teachers believe their abilities to deal with disruptive behavior in class and create a classroom environment suitable for PjBL. *SES* represents the extent to which mathematics teachers believe their abilities to create PjBL tasks to evoke students' learning interest and help students learn mathematics through PjBL.

P-MPCK refers to the knowledge and/or ability to carry out PjBL in math. Similar to the cognitive facets in the modeling framework (Borromeo Ferri, 2018), P-MPCK could be specified into four dimensions:

1. The theoretical dimension provides a background for PjBL in mathematics education, including conceptual knowledge of PjBL (Ministry of Education, 2022; Thomas, 2000, e.g., theoretical foundation and curriculum demand) and the values and educational aims related to PjBL (Herro et al., 2021; Kokotsaki et al., 2016; e.g., enhancing students' collaborative problem-solving skills), namely, *knowledge about concepts and aims* (KCA).
2. The task dimension refers to the ability to analyze and create PjBL tasks; thereby it includes knowledge about the characteristics and evaluation criteria of PjBL tasks (Markham et al., 2003; e.g., challenging questions, sustained inquiry, and public product), namely, *knowledge about characteristics of PjBL* (KCH).
3. The diagnostic dimension represents the ability to identify phases in PjBL lessons (Fisher et al., 2020; e.g., entry, inquiry, and presentation phase) and to diagnose students' difficulties and misconceptions with their causes (Thomas, 2000; e.g., the challenges in generating meaningful scientific questions and conducting systematic inquiry activities), namely, *knowledge about PjBL processes* (KDG).
4. The instruction dimension focuses on interventions when implementing PjBL lessons, which contains different intervention measures along with corresponding effects on students' thinking and behavior (Thomas, 2000; e.g., providing scaffolds for learning or suggestions on teamwork), namely, *knowledge about interventions* (KIT).

METHOD

Participants

The data used in this study was partly collected from the "in-service teacher training program towards core competence". From 2019, our team has carried out multiple training projects in middle schools from Beijing, Shanghai and other cities in China, in order to foster a group of teachers competent to design and implement PjBL in math and promote teacher professional development. Training and guidance were generally conducted through lesson studies (Fang et al., 2022), where each project contained four fundamental stages (study, planning, implementation, and reflection), and five main aspects (general training, lesson preparation, trial teaching, formal teaching, and expert guidance). For each project, the team consisted of diverse individuals, with one teacher as the main designer and implementer of PjBL, and other colleagues, educational researchers, and university researchers serving as guides or collaborators, who worked together to complete the whole instructional design. These teachers independently designed or collaborated on unit projects and submitted their proposals.

112 participants in our practice-based teacher-training program on PjBL (PB program) completed the web-based questionnaire. To ensure the applicability of the questionnaire, 234 teachers who attended other theory-oriented teacher-training programs on PjBL (TB program) were surveyed.

Table 1. Scales of teacher beliefs and teacher self-efficacy

Scale	Subscale	Number	Sample (to what extent do you agree with the following statements?)
Teacher beliefs	BFP	8	" PjBL is beneficial to enhance students' creative thinking and practical ability."
	CBF	4	"Teachers should allow students to propose their unique solutions to problems."
	TBF	4	"Teachers should provide detailed procedures for students to imitate and practice."
Teacher self-efficacy	SEI	8	"I am competent to propose challenging questions for high-level students."
	SEC	8	"I am competent to establish activity rules of group work."
	SES	8	"I am competent to create PjBL tasks to evoke students' learning interest."

Instruments

To ensure the content validity of the assessment instrument, the following steps were taken. First, a PhD student majoring in mathematics education translated the modeling instrument from English into Chinese. Second, the first author replaced or deleted items related to mathematical modeling by PjBL-related ones based on our theoretical framework. Then, a focus group including one professor, one assistant researcher, and four PhD students, discussed about how to modify the ambiguous expressions and revise inappropriate items; for instance, "the achievements of PjBL have social value" was changed to "the products or proposals that students developed have social value after solving PjBL tasks". Subsequently, six experts (three middle school mathematics teachers with rich experience related to PjBL and three teacher educators focusing on PjBL) were asked whether they understood each item and to answer the whole questionnaire, and then modifications were further made according to their feedback. As for inconsistent responses to close-ended items (combined-true-false items and multiple-choice items), authors set the response that either was consistent with the initial intention from a focus group or at least two out of three experts chose, as the correct one. Finally, a questionnaire was developed to capture mathematics teachers' professional competence in PjBL contexts. Teacher beliefs and teacher self-efficacy were measured via five-point Likert scales (see **Table 1**) with the ratings *totally disagree, mostly disagree, neutral, mostly agree, and totally agree*; P-MPCK was assessed via combined-true-false items and multiple-choice items.

In the scale of teacher beliefs, 16 items were totally included and divided into three dimensions. Some items were revised from the modeling instrument through changing the modeling-specific beliefs into the PjBL-specific beliefs, while others related with mathematics teaching and learning were retained.

Regarding teacher self-efficacy, each of three subscales contained 8 items to measure the self-efficacy of applying instructional strategies, dealing with learning difficulties and disruptive behavior in the classroom, and motivating students' interest in mathematics in PjBL contexts. All 24 items were revised from ratings on a nine-point scale (Tschannen-Moran & Hoy, 2001) to those on a five-point scale. The question formats were changed for the convenience of response and the consistency with scales of teacher beliefs, for instance, item "to what extent can you craft good questions for your students?" was changed to "to what extent do you agree with that *I am competent to craft good questions for students*?" Moreover, items with the lowest loadings or unrelated to PjBL were replaced by new ones, for example, item "to what extent can you make your expectation clear about student behavior?" was removed, and item "to what extent do you agree with that *I am competent to articulate the activity requirements related to PjBL*?" was added.

P-MPCK was divided into four dimensions. To measure KCA and KCH, each of two dimensions contained ten positive and two negatively worded items for discriminating responses so that 24 combined-true-false items were developed. For instance, item "please check whether the following statement is true or false: The PjBL tasks come from students' needs." is a negatively worded item; that is, if a teacher chose the "true" option, it means that he/she was wrong because the PjBL tasks usually origin from real-world problems, but do not necessarily come from students' needs. KDG and KIT were measured through 12 multiple-choice items based on three text vignettes in real PjBL scenarios, which was seen as the "situated approach". Here, four multiple-choice items for each scenario were created, with the first two items for the diagnosis of difficulties and causes and the last two items for intervention measures and potential effects. Compared with video-based tests for noticing (Kaiser et al., 2015), students' dialogues for completing different PjBL tasks were presented through text vignettes because they are cognitively less burdensome and convenient online (Yang et al., 2022; Zeeb et al., 2023). Similar to the modeling instrument, an upstream general scenario was created to specify the purpose of the dialogues and items, the teaching experience of teachers, the students' level of performance, and the specific requirements. For each task, an individual introduction to the PjBL-specific context was presented in text-vignette format. Task 1 was to recognize the difficulty in data collation along with the potential cause and intervention measures during an entry event. Task 2 explored the evaluation criteria and assigned different weights to each evaluation dimension in the final presentation phase. Task 3 aimed at identifying difficulties, causes, and solutions related to task allocation during the inquiry phase. **Figure 2** illustrates project background on task 1 and a discussion of students in a group. Sample items are

Task 1: Household white waste

Background: Based on "Statistics and Probability" module in Grade 7, this project aims to introduce the activity of "making electronic posters" to enable students to use statistical surveys to understand the types and daily disposal methods of household white waste, and provide rational suggestions for reducing household white waste.

Students' Dialogue:

S1: Did you receive feedback on the questionnaire sent yesterday?

S2: I have received 10 copies, but haven't finished organizing them yet. The responses vary widely. One copy even counted all the white items of trash at home as 'white garbage'. The ways to waste disposal mentioned are quite diverse as well.

S3: What questions were included in the survey?

S4: The name of their household white waste and how they usually handle it. The proportion of household white garbage to the total waste was calculated by themselves.

Figure 2. Text vignette for task 1 (Source: Original task by author)

1.1. Which is the largest difficulty that these students are currently facing?

A. data collection B. data collation

C. data analysis D. statistical chart drawing

1.2. Which is the most likely reason for the current difficulty?

A. inappropriate respondents B. improper survey design

C. unclear objective of survey D. inappropriate method of survey

1.3. If you were the teacher, which is the most suitable intervention to address the current difficulty?

A. "Please recall what is the original intention of our questionnaire survey."

B. "Where else can we find more respondents?"

C. "Can we set options for the questionnaire based on the preliminary research?"

D. "Is there a more suitable method of survey?"

1.4. After intervention, what are these students most likely to do?

A. Place the objective of survey in the questionnaire description

B. Make interviews with familiar friends or classmates

C. Increase samples through issuing questionnaires at the entrance of their community

D. Collect information on the types of household white waste and the ways to waste disposal

Figure 3. Sample items for task 1 (Source: Original task by author)

specified in **Figure 3**, where item 1.1 and item 1.2 correspond to KDG, and item 1.3 and item 1.4 correspond to KIT.

Data Analysis

The respondents were divided into two groups (sample A and sample B). Each group included 56 teachers who participated in the PB program and 117 teachers who participated in the TB program. Sample A was used for item analysis and reliability analysis, whereas sample B was used for validity analysis.

After data cleaning, the questionnaire was validated using SPSS 27.0. For teacher beliefs and teacher self-efficacy, Cronbach's alpha and split-half reliability were used to capture the reliability of (sub-)scales. After that, we examined the construct validity of scales by exploratory factor analysis (EFA) instead of confirmatory factor analysis (CFA), because these scales were utilized to measure teacher competence in PjBL contexts for the first time, and larger sample size was required by CFA (Charalambous et al., 2008).

For P-MPCK, the reliability of items was tested in a dichotomous Rasch model based on item response theory (IRT) that is suitable for estimations of an individual's ability or the trait level independently of other items (Adams et al., 1997). The simple Rasch model was utilized to scale dichotomous items and estimate item quality and person-ability parameters in the ACER ConQuest software. Different from scales, the construct validity of non-scale data (knowledge facet) theoretically supported, was usually tested through CFA (Flora & Curran, 2004). However, in order to ensure the valid conceptual model given our smaller sample size of N = 173 teachers, we summed the corresponding item scores in each dimension of P-MPCK as parcel scores, which could reduce the estimated parameters to 4 parcels (Krauss et al., 2008).

For three facets of mathematics teachers' professional competence in PjBL contexts, their correlations were calculated to examine the convergent and discriminant validity of questionnaire (Wess et al., 2021).

RESULTS

Reliability

In terms of reliability test (**Table 2**), Cronbach's alpha coefficients of teacher beliefs and teacher self-efficacy scales were 0.88 and 0.94, respectively, and Cronbach's alpha coefficients of each subscale reached over 0.8, implying good stability and internal consistency. Spearman-Brown coefficients of each (sub-)scale ranged from 0.81 to 0.96, indicating a good split-half reliability. The EAP reliability score for P-MPCK items calculated based on IRT was comparable to Cronbach's alpha coefficient in classical test theory (Adams, 2005). Results show that reliability scores for P-MPCK ranged from 0.60 to 0.90, indicating acceptable or good reliability for a statistical analysis.

Construct Validity

For the integrated scale of teacher beliefs and teacher self-efficacy, KMO value (0.88) and Bartlett's test (p < 0.01) indicated that these items were suitable for factor analysis. More than 70% of the total variance in the integrated scale was explained by six factors (**Table 3**), which was consistent with the conceptual model of mathematics teachers' competence in PjBL contexts.

According to the EFA, eight items were deleted because they had a factor loading below 0.5 or loaded on an unexpected factor. For P-MPCK, five combined-true-false items and one multiple-choice items were removed due to insufficient discrimination. Finally, 30 items for the knowledge tests and 32 items for two scales were retained.

Table 2. Reliability for the assessment instrument

Facet	Number	Cronbach's alpha	Split-half reliability
Teacher beliefs	16	0.88	0.93
BFP	8	0.94	0.95
CBF	4	0.84	0.86
TBF	4	0.80	0.81
Teacher self-efficacy	24	0.94	0.96
SEI	8	0.89	0.91
SEC	8	0.88	0.92
SES	8	0.92	0.93
P-MPCK	36	0.86	-
KCA	12	0.76	-
KCH	12	0.90	-
KDG	6	0.64	-
KIT	6	0.60	-

Table 3. Factor loadings of the integrated scale of teacher beliefs and teacher self-efficacy

Item	F1	F2	F3	F4	F5	F6
BFP5	0.903					
BFP6	0.879					
BFP7	0.868					
BFP8	0.850					
BFP4	0.835					
BFP2	0.827					
BFP1	0.770					
BFP3	0.703					
CBF7		0.774				
CBF6		0.713				
TBF4			0.816			
TBF8			0.745			
TBF3			0.745			
SEI3				0.745		
SEI4				0.739		
SEI2				0.737		
SEI1				0.668		
SEI5				0.623		
SEI6				0.550		
SEC3					0.826	
SEC4					0.745	
SEC1					0.740	
SEC5					0.676	
SEC2					0.671	
SES3						0.828
SES7						0.798
SES4						0.782
SES5						0.767
SES8						0.733
SES6						0.656
SES2						0.642
SES1						0.633

Note. F: Factor

CFA results suggest that the knowledge facet in PjBL contexts could be measured by four-dimension model of P-MPCK, because all model fit indices were within the reference interval (χ^2/df = 1.251, RMSEA = 0.043, GFI = 0.995, CFI = 0.999, NFI = 0.994, TLI = 0.993).

Table 4. Correlations of four dimensions of P-MPCK

	KCA	KCH	KDG	KIT
KCH	0.78**			
KDG	0.43**	0.24**		
KIT	0.36**	0.14*	0.61**	
P-MPCK	0.88**	0.73**	0.74**	0.70**

Note. *$p < 0.05$ & **$p < 0.01$

Table 5. Correlations of P-MPCK with teacher beliefs and teacher self-efficacy

Teacher beliefs	P-MPCK	Teacher self-efficacy	P-MPCK
BFP	0.59**	SEI	0.14*
CBF	0.48**	SEC	0.11*
TBF	-0.17*	SES	0.08

Note. *$p < 0.05$ & **$p < 0.01$

Convergent and Discriminant Validity

For convergent validity of the knowledge test instrument, intercorrelations of the four dimensions of P-MPCK (**Table 4**) were calculated. A larger correlation coefficient indicates higher convergent validity and lower discriminant validity. As expected, P-MPCK correlated significantly with the four dimensions (r > 0.7), indicating they can capture teachers' cognitive facets in PjBL contexts. However, there were weak correlations (r < 0.45) of KCH (or KCA) with KDG and KIT, indicating that professional knowledge measured by situated approach significantly differed from other knowledge facets.

To test discriminant validity of the whole questionnaire, we also calculated correlations between P-MPCK and the other two facets of teacher competence. As shown in **Table 5**, P-MPCK was distinguished from teacher self-efficacy (r < 0.2). The negative correlation between P-MPCK and TBF and the significantly positive correlations between P-MPCK and BFP and CBF indicate that mathematics teachers with higher P-MPCK tend to reject transmissive theories of learning and teaching practices and favor the constructivist perspectives.

DISCUSSION AND CONCLUSIONS

The main contribution of this study is that we proposed a theoretical framework of mathematics teachers' competence in PjBL contexts and developed a well-validated assessment instrument. Drawing on extensive prior literature, the theoretical framework provides a sound foundation for the development of conceptual models and the adaptation of assessment instruments.

The present study aimed to construct a conceptual model of mathematics teachers' competence in PjBL contexts, which was challenging work. As reviewed above, little research has a focus on mathematics teachers' competence in PjBL contexts. In China, relative studies ignored the affective-motivational facets and subject-related features of teacher competence. To fill the research gap, we adopted a modeling framework with three facets (teacher knowledge, beliefs, and self-efficacy) and their dimensions as a reference given the similarities between mathematical modeling and PjBL. The modeling framework has been validated scientifically and empirically, and items from the modeling instrument and other related instruments could be revised and updated for our study. Cronbach's alpha coefficients and EAP reliability scores were used to test the reliability of scales and knowledge facet, respectively. The content validity was assured by discussions in a focus group and expert opinions. The approaches of factor analysis were employed to examine the construct validity of conceptual model. To further guarantee the validity of model, convergent and discriminant validity were tested through correlations among (sub-)facets of teacher competence. Consequently, it is reasonable to conclude that the model could be applied to conceptualize professional competence of mathematics teachers in PjBL contexts.

Indeed, the assessment instrument was developed in order to investigate the effectiveness of teacher education related to PjBL, so we focused on teachers' knowledge and skills in PjBL contexts relatively. Apart from cognitive approach, a situated approach with text vignettes was used to test mathematics teachers' abilities to diagnose students' difficulties and misconceptions and adopt interventions to help students learning in PjBL contexts, which were similar to noticing skills (Blömeke et al., 2015; Kaiser et al., 2015). Compared with video-based tests for noticing skills, text-based tests are cognitively less burdensome and convenient online (Yang et al., 2022; Zeeb et al., 2023). The text vignettes came from real PjBL scenarios, which could be replaced based on the measure demands. Therefore, we consider that the assessment instrument integrated cognitive approach with situated approach has the potential for a holistic understanding of teacher competence and could be popularized to other contexts.

Nevertheless, some shortcomings in this work should be pointed out. First, there were limitations in data collection and differences in data types (five-point Likert scale and close-ended questionnaire), so it was difficult to validate the goodness of fit of the whole conceptual model. With the increase in samples and the development of analysis methods, more rigorous validation of a conceptual model will be carried out. In addition, professional competence to teach PjBL in math is regarded as a complicated construct with three facets, but the conceptual model and the assessment instrument will be advanced based on a more nuanced understanding of teacher competence in PjBL contexts. On the one hand, the increased need for teaching Artificial Intelligence in class will facilitate the integration of technological pedagogical content knowledge into cognitive facet (Kim et al., 2021). On the other hand, if time is sufficient and video technique is accessible, using video to assess teachers' ability to noticing will be an important complement to the high-quality measurement instrument of teacher competence in teacher education (Weyers et al., 2023)

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