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Teaching and learning processes for pre-service mathematics teachers: The case of systems of equations

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Abstract

This study aims to explore the teaching and learning processes of the school mathematics course for pre-service mathematics teachers in the case of systems of equations. Types of teaching and learning approaches and a symbol sense perspective were used as theoretical frameworks for this study. To do so, we conducted a qualitative case study, involving 35 pre-service mathematics teachers, in the form of classroom observations in one of the state universities in Bandung, Indonesia. The observations included the teaching and learning processes, a written formative test, and interviews. The results showed that the teaching and learning processes are implemented by using a deductive approach, and the written student work from the formative test and interviews revealed various solution strategies. From the symbol sense perspective, we conclude that the teaching and learning processes need to be enhanced to promote pre-service teachers' procedural skills and conceptual understanding in a balanced manner.

Keywords: pre-service mathematics teachers, symbol sense, systems of equations, teaching and learning approaches

INTRODUCTION

The system of equations is one of the algebra topics taught in school mathematics (SM) for secondary school students all over the globe (e.g., Akpalu et al., 2018; Jupri et al., 2020; Kirvan et al., 2015; McCrory et al., 2012; Proulx et al., 2009). This topic is often difficult not only for secondary school students (Proulx et al., 2009; Van Amerom, 2003) but also for mathematics education students as pre-service mathematics teachers (Kılıç, 2011), including in Indonesia (Dewi et al., 2021). The difficulties in solving systems of equations encountered either by secondary school students or pre-service mathematics teachers include, among others, selecting more efficient solution strategies, avoiding algebraic manipulation and calculation errors, and forgetting to check whether solutions satisfy the initial equations within the systems or not (Dewi et al., 2021; Proulx et al., 2009). Other difficulties in solving algebra problems encountered by pre-service mathematics teachers include, for instance, passing through from arithmetical to algebraic thinking in solving word algebra problems (Van Dooren et al., 2003), translating word problems into algebraic models (Isik & Kar, 2012), and using systems of linear equations to solve proportion problems (Irfan et al., 2019). These difficulties to a certain extent are the impact of the teaching and learning processes. Relevant studies on the teaching and learning process for pre-service mathematics teachers showed, inter alia, that the teacher education program is focused more on knowledge acquisition than on engaging in teaching and learning (Chamoso et al., 2012), and that the pre-service mathematics teachers were inadequate in terms of knowledge of students' algebraic concepts (Tanisli & Kose, 2013). For future careers of pre-service mathematics teachers, these difficulties should be overcome. In the Indonesian context, an effort to do so is by strengthening pre-service mathematics teachers' procedural skills and conceptual understanding in various mathematics courses.

SM is one of the courses for pre-service mathematics teachers in Indonesia (Jupri et al., 2022). The system of equations is one of the topics addressed in this course. In the SM course, each mathematics topic is addressed by focusing on strengthening procedural skills, conceptual understanding, and its applications. Regarding this important course, we would like to know how the

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Contribution to the literature

- This study provides a clear description of the teaching and learning processes for pre-service mathematics teachers in the Indonesian context for the case of systems of equations.
- This study presents the results of an analysis of the teaching and learning processes for pre-service mathematics teachers using a framework of deductive and inductive teaching approaches.
- This study addresses the impact of the teaching and learning processes on pre-service mathematics teachers' conceptual understanding and procedural skills for the case of systems of equations from a symbol sense perspective.

teaching and learning processes are implemented for pre-service mathematics teachers, particularly for the topic of systems of equations.

To delve into the teaching and learning processes for the SM course, we carried out qualitative research in the form of classroom observations for the case of solving systems of equations. This type of investigative research, particularly for comprehending the teaching and learning processes of pre-service mathematics teachers in Indonesia, to a certain extent is still rarely carried out (e.g., Jupri et al., 2022). Therefore, the present research aims to explore the teaching and learning processes of the SM course for pre-service mathematics teachers and their effect, particularly on the ability to solve systems of equations.

THEORETICAL FRAMEWORK

We use types of teaching approaches and the perspective of symbol sense as theoretical frameworks to investigate the teaching and learning processes and their impact on pre-service mathematics teachers' ability in dealing with systems of equations. The types of teaching are used for analyzing the teaching and learning processes, and the symbol sense perspective is for analyzing pre-service teachers' ability to solve systems of equations.

Types of Teaching Approaches

According to Prince and Felder (2006), in general, there are two types of teaching approaches, i.e., inductive and deductive teaching approaches. A deductive teaching approach is a teaching approach that applies deductive thinking in the teaching and learning processes, i.e., teaching mathematical ideas from general to more specific cases (Jupri et al., 2021; Ndemo et al., 2017; Prince & Felder, 2006; Young, 1968). Using this approach, therefore, the teaching and learning process is carried out, respectively by explicating definitions, concepts, and principles; using them in solving exemplified problems; providing exercises for students and corresponding classroom discussion; and conducting an individual written formative test. As the teaching process is dominantly played by the teacher, we can consider that the deductive teaching approach can be classified as a teacher-centered approach to teaching (Ramsden, 1987; Stephan, 2020).

An inductive teaching approach is a teaching approach that applies inductive thinking in the teaching and learning processes, i.e., teaching mathematical ideas from specific to more general cases (Jupri et al., 2021; Ndemo et al., 2017; Prince & Felder, 2006; Young, 1968). In this way, the teaching process is implemented, respectively by posing specific problems for explorations; constructing conjectures, concepts, principles, or formulas through solving the problems; applying the concepts, principles, or formulas for solving problems; and drawing general conclusions. As the teaching process is dominantly played by students actively, the inductive teaching approach can be classified as a student-centered approach to teaching and learning (Byusa et al., 2020; Ramsden, 1987; Stephan, 2020).

Symbol Sense

Even if the idea of symbol sense is difficult to define precisely, it can be described as an intuitive feel for when to use symbols and when to ignore them in the process of solving a mathematical problem (Arcavi, 1994). This notion, as an analogy to number sense, can be recognized as an ability to understand and perceive important structures to symbols, mathematical expressions, formulas, equations, and systems of equations (Arcavi, 2005). Symbol sense characteristics that are useful for solving systems of equations include, among others, the ability to utilize symbols in recognizing relationships; the skill to read through and manipulate symbolic expressions; the skill to verify the symbol meanings in the implementation of a procedure, the solution of a problem, or during the inspection of a result; the capability of understanding symbolic relationships that express graphical information; and the realization that symbols can play roles as variables or parameters (Arcavi, 2005; Jupri et al., 2022; Kop et al., 2020).

The perspective of symbol sense has been used in several previous research, for instance, for comprehending student difficulties and understanding on the concept of the parameter (Drijvers, 2000); and for investigating student understanding in dealing with equations and algebraic expressions in a digital environment (Bokhove & Drijvers, 2010, 2012); and in case of Indonesia, for comprehending students' algebraic reasoning in solving substitution problems (Jupri et al., 2016) and solving absolute value equations and inequalities (Jupri et al., 2022).

Having symbol sense characteristics when dealing with symbols is considered to be one of the most important aspects of the success of learning algebra (Bokhove & Drijvers, 2010; van Stiphout et al., 2013). The success in learning algebra by having symbol sense characteristics shows an algebraic proficiency, which signifies a relational understanding rather than only an instrumental understanding (Skemp, 1976). Algebraic proficiency can be seen as having aspects of conceptual understanding and procedural fluency in symbolic representation (Brown & Quinn, 2007; van Stiphout et al., 2013). Conceptual understanding concerns an understanding of mathematical concepts, relations, and operations; and procedural fluency is interpreted as the skill of implementing mathematical procedures efficiently, flexibly, accurately, and appropriately (Kilpatrick, 2001). These two aspects of proficiency have to go hand in hand in supporting proficiency in algebra and in improving algebraic expertise in particular. Algebraic expertise, which can be interpreted from the symbol sense perspective, is seen as an algebraic ability that ranges from basic skills to strategic work (Bokhove & Drijvers, 2010; 2012; Drijvers et al., 2010). Basic skills include procedural work with a local focus and algebraic manipulation, and strategic work requires a global focus and algebraic reasoning and conceptual understanding. For the present study, the framework of symbol sense is used for comprehending pre-service mathematics teachers' understanding in dealing with systems of equations.

RESEARCH METHODS

Design of Study

We conducted a qualitative case study through classroom observations to investigate the teaching and learning processes of the SM course and its impact on pre-service mathematics teachers' ability to deal with systems of equations (Yin, 2016). To do so, two stages of observation were carried out. In the first stage, we observed the teaching and learning processes that were implemented in two meetings, which lasted for 2 x 150 minutes, involving 35 pre-service mathematics teachers from one of the state universities in Bandung, Indonesia. In the second stage, we observed an individual formative written test on solving systems of equations, which lasted for 100 minutes, and conducted interviews with six selected pre-service mathematics teachers.

Participants

This qualitative case study involved 35 students (28 females and 7 males) of a mathematics education program, as pre-service secondary mathematics teachers (20-21 year-olds), from one of the state universities in Bandung, Indonesia. These students were in the third semester of a mathematics education program. They had studied some essential mathematics courses, such as foundations of mathematics, differential and integral calculus, number theory, Euclidean geometry, analytical geometry, and statistics. The first author taught an SM course to them in the third semester, in which the system of equations is one of the topics within this course. As the topic of systems of equations was already taught at secondary school levels, therefore, the teaching of these topics is to deepen pre-service teachers' procedural skills and conceptual understanding. Taking this background into account, we consider that they have sufficient mathematics skills and knowledge to take part in the current study.

Data Collection

Based on the stages of the present study, we did the following data collection. First, we collected data about the steps of teaching and learning processes from two meetings (2 x 150 minutes) of the SM course, including lecture notes, pictures of teaching situations, and student responses on the topic of systems of equations. This topic includes systems of linear equations in two and three variables and systems of non-linear equations in two and three variables. The data were collected through an observation sheet and field notes. The observation sheet contains spaces for taking notes on the opening session of the lesson, main session (such as classroom exercise and discussion) and closing session. Field notes contain blank written spaces for taking notes on activities relevant to the teaching and learning processes, such as comments on student responses and comments on student work during the classroom discussion.

Second, we administered an individual written formative test on solving systems of equations after the teaching and learning processes, involving 35 pre-service mathematics teachers, which lasted for 100 minutes. During the test, the participants were not allowed to use smartphones or other electronic devices. **Table 1** presents five task items for the written test, in which each item concerns a different type of system of equations: A system of linear equations in two variables; a system of linear equations in three variables; a system of non-linear equations in two variables; a system of non-linear equations in two variables (having linear-quadratic forms); and a system of non-linear equations in three variables (having multiplication forms).

Third, we conducted interviews with six pre-service mathematics teachers to verify their thinking for the use of particular solution strategies to systems of equations, and to clarify their unclear written work from the written test. The six interviewees were chosen based on their written work that represents different solution strategies for solving systems of equations. For doing the interview

Table 1. Systems of equations tasks for formative test

No	Tasks	Systems of Equations
1.	A system of linear equations in two variables	If the system of equations below has the solution $(x, y) = (1, 3)$, then $a - b = \cdots$ $\begin{cases} ax + by = 11 \\ 7x - by = 1 \end{cases}$.
2.	A system of linear equations in three variables	Consider the system of equations below. $\begin{cases} 3x + 2y + z = 7 \\ x + 3y + 2z = 3 \\ 2x + y + 3z = 2 \end{cases}$. Find $x + y + z = \cdots$
3.	A system of non-linear equations in two variables	Consider the system of equations below. Find $x + y = \cdots$ $\begin{cases} \frac{4}{x} + \frac{1}{y} = p \\ \frac{5}{x} - \frac{2}{y} = q \end{cases}$
4.	A system of non-linear equations of two variables (having linear-and-quadratic forms)	Find the solution to the system of equations below. $\begin{cases} y = 2 - x \\ y = (x - 2)(x + 3) \end{cases}$
5.	A system of non-linear equations in three variables (having multiplication forms)	Consider the system of equations below. $\begin{cases} x + y + xy = 39 \\ x + z + xz = 29 \\ y + z + yz = 47 \end{cases}$. Find $2x - y + z = \cdots$

we used an interview guideline. In this guideline, we used non-intervening questions to clarify their unclear work and to ask reasons for the use of solution strategies. The questions for the interviews include, for instance, asking participants whether they are making sense of a system of equations before solving it, asking participants to explain their written work, asking participants to provide reasons for the solution strategies they chose; and asking whether they are checking the solutions they found for each system of equations or not.

Data Analysis

We did the data analysis, as follows. For the data about the teaching and learning processes, we used the framework of types of teaching approaches. In this analysis, we scrutinize procedures of the teaching and learning processes, teaching and learning contents for the topic of systems of equations, and pre-service teachers' responses during the teaching and learning processes.

For analyzing the written work and interview data, we used the symbol sense perspective as a framework. In this analysis, we identified strategies used by the participants for solving systems of equations and identified participants' difficulties during the solution processes. We classified a system of equations solving strategy into two: a symbol sense strategy if a student uses symbol sense characteristics, and a procedural strategy if a student does not use symbol sense characteristics. These identifications led us to conclude whether the students have acquired both conceptual understanding and procedural skills sufficiently. Transcribed interview data was analyzed using a symbol sense perspective to clarify or strengthen the written work data. In particular, we used the perspective of symbol sense to comprehend the selection of a particular strategy for solving systems of equations.

RESULTS

In this section, we present the results of the two stages of observations: The teaching and learning processes and written work with corresponding interviews from the individual formative test for the case of solving systems of equations.

Teaching and Learning Processes for the Case of Systems of Equations

As the topic of systems of equations has been studied at the secondary school level, the teaching and learning processes were started by the lecturer by asking mathematics education students, as pre-service teachers, about the meaning of a system of equations. Several students tried to respond to the question. After discussing some of the student responses through questions and answers, the lecturer concluded that a system of equations refers to a collection of two or more equations that represent conditions that must be satisfied simultaneously by each equation. The system is not a collection of independent equations, but a set of equations that are in relationship. To clarify the definition, the lecturer then provided examples of systems of equations, including systems of linear equations and systems of non-linear equations.

The lecturer proceeded to address systems of linear equations in two variables. First, he wrote down the general form of the system of linear equations in two variables. Next, he gave the following system as an

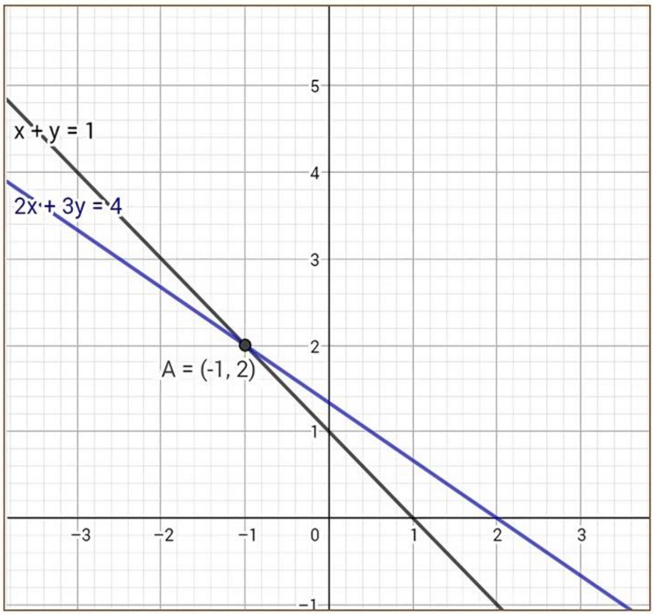


Figure 1. The graphs of $x + y = 1$ and $2x + 3y = 4$ (Source: Authors' own elaboration)

example: $\begin{cases} x + y = 1 \\ 2x + 3y = 4 \end{cases}$ … (I). To give a visual meaning of this system, through questions and answers, the lecturer then showed students how to solve it using a graph method with the help of GeoGebra software (shown in **Figure 1**). When asking a question, the lecturer would continue the teaching process if the students gave relevant answers. The solution of the system, geometrically, is the intersection point between the two equations, namely $A = (x, y) = (-1, 2)$. Therefore, $x = -1$ and $y = 2$ satisfy the system and as such be the solution to the system. The lecturer emphasized that when a solution is found, we should check whether it satisfies each equation in the system or not.

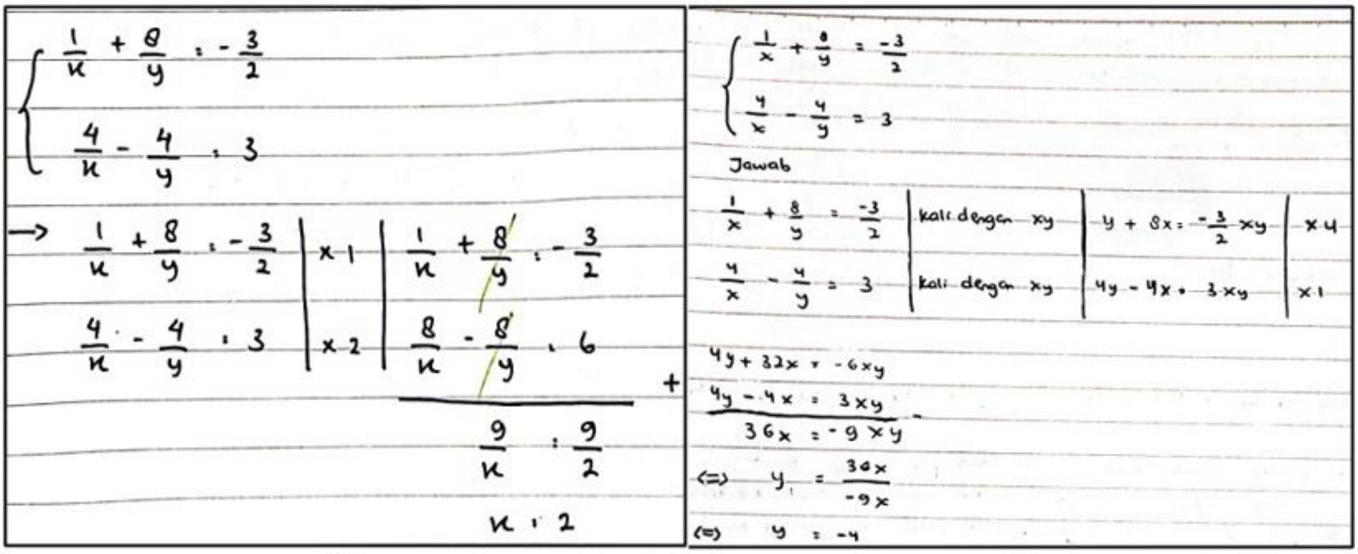
After explaining the graph method, the lecturer explained how to solve the system consecutively by the elimination method, by the substitution method, and by the combination of elimination and substitution methods. In this teaching process, the lecturer enthusiastically explained each method completely and comprehensively. For instance, using the combination method, the system (I) is solved by the lecturer, as follows. By multiplying the first equation in (I) by 3, and subtracting the result from the second equation, we obtain $x = -1$. This part concerns the elimination method. After that, by substituting $x = -1$ into the second equation, we obtain $2(-1) + 3y = 4$, which leads to $y = 2$. This means that the solution of the system (I) is $x = -1$ and $y = 2$. The system of linear equations in three variables was addressed similarly to the systems of linear equations in two variables.

Table 2 presents the first exercise (exercise 1) that was given by the lecturer. Students were given an opportunity for about 15-20 minutes to solve some of the tasks regarding systems of linear equations in two and three variables. Some of the tasks that are solved by the students were discussed. Some other tasks that were not discussed in the classroom discussion were used as homework. In this exercise, task E1.1(c) and task E1.1(e) concern systems of non-linear equations that can be changed into systems of linear equations forms during the solution processes. In this first exercise, task E1.1(c) and task E1.1(d) were addressed in the classroom discussion.

Table 2. Exercise tasks on systems of equations in two or three variables

Exercise 1. Solve each task below!
E1.1. Find the solution set for each system of equations below! (a) $\begin{cases} 7x - 3y = 13 \\ 3x + 5y = -7 \end{cases}$. (b) $\begin{cases} \frac{x+y-2}{5} + \frac{x-y+1}{4} = -3 \\ \frac{x+8}{2} + \frac{y}{3} = 2 \end{cases}$. (c) $\begin{cases} \frac{1}{x} + \frac{8}{y} = -\frac{3}{2} \\ \frac{4}{x} - \frac{4}{y} = 3 \end{cases}$ (d) $\begin{cases} x + y - z = -3 \\ 2x + y + z = 4 \\ x + 2y + z = 7 \end{cases}$. (e) $\begin{cases} \frac{4}{x} + \frac{2}{y} + \frac{3}{z} = 1 \\ \frac{4}{x} + \frac{4}{y} + \frac{3}{z} = 2 \\ -\frac{8}{x} + \frac{2}{y} - \frac{6}{z} = 1 \end{cases}$
E1.2. Ten years ago, Andi's age was twice Budi's age. Five years later, Andi's age will be 1 ½ times Budi's age. How old are Andi and Budi now?
E1.3. The perimeter of a rectangle is 70 cm. If the length is doubled and the width is 1/3 of the original width, then the perimeter of the rectangle is 90 cm. Find the length and width of the original rectangle.
E1.4. A number consists of three digits. The sum of the three digits is equal to 9. The value of the number is equal to 14 times the sum of the three digits. The third digit minus the second digit and the first digit equals 3. Find the number!

During the classroom discussion, the lecturer provided an opportunity for students to present different solution strategies for the same task. For example, **Figure 2** presents two different solution strategies for solving task E1.1(c) taken from written student work before they were presented on the board in front of the class. Part a in **Figure 2** shows the use of the combination method: The elimination method was used by eliminating the term $\frac{8}{y}$ directly to obtain $x = 2$, and the substitution method was used by substituting $x = 2$ into one of the equations to get $y = -4$. Part b in **Figure 2** also presents the use of the combination methods in a different way. Before applying the elimination method, the student multiplied each of the equations in the system using the term xy. When addressing the solution to this task during the classroom



(a) (b)

Figure 2. Two different strategies for solving task E1.1(c) (Source: Authors' own elaboration)

discussion, one of the students (student 1) commented and thought that it was more complicated and subtle to use the first strategy than the second strategy (part b in **Figure 2**). To respond to this opinion, the lecturer did not directly provide his opinion, but he asked other students to respond with their opinion, as shown in the excerpt below.

> Student 1: Sir, in my view the first strategy (part a in **Figure 2**) is more difficult than the second strategy (part b in **Figure 2**). Using the first strategy, it is difficult to see common terms for the elimination method. Using the second strategy, however, we can multiply both sides using the terms xy and next we can simply use the elimination method to solve. What would be your opinion and suggestions for this case?
>
> Lecturer: Good question! Okay, can anyone respond to this question?
>
> Student 2: I do not think I agree with her (student 1). In my view, the first strategy is easier because we can assign, for instance, $\frac{1}{x} = p$ and $\frac{1}{y} = q$. So, we can obtain a system of linear equations in two variables. The system can be solved easily using the elimination or substitution method.
>
> Student 3: I agree with student 2's opinion. Yes, I think the first strategy is easier to use than the second one.
>
> Student 1: Hmm … [She is thinking for moments in a few minutes]. Aha … I understand! The first strategy is easier to use when we assign the terms using other variables! But still, the second strategy is also easy because we can directly use the elimination method after the procedure of multiplying both sides of equations using the xy term.

After the classroom discussion, the teaching and learning process was stopped and would be continued in the next meeting.

In the second meeting, in general, the lecturer continued the teaching processes by addressing the systems of non-linear equations, providing corresponding exercises, and addressing the homework of the previous meeting in the classroom discussion. First, the lecturer explained systems of non-linear equations for the case of two equations consisting of linear and quadratic functions. After writing down the general form for the system and explaining it, the lecturer then gave the following system as an example: $\begin{cases} y = 2x - 1 \\ y = x^2 - 3x + 5 \end{cases}$ … (II). Using the substitution method, the lecturer obtained $2x - 1 = x^2 - 3x + 5$, which leads to $x = 2$ or $x = 3$. By substituting these x values into one of the equations in the system, then it can be obtained $y = 3$ or $y = 5$. Therefore, the solution of the system is $(x, y) = (2, 3)$ or $(x, y) = (3, 5)$.

Similar to the first meeting, during the process of explanation, the lecturer invariably uses a questions and answers strategy, in which he would continue if there were an appropriate response to his question. Next, the lecturer explained the use of the graph method to solve the system (II). He used GeoGebra software as the tool for drawing graphs of $y = 2x - 1$ and $y = x^2 - 3x + 5$. The solution of the system (II), as can be seen in **Figure 3**, is the intersection points between the two equations, namely $(2, 3)$ or $(3, 5)$. Therefore, $(x, y) = (2, 3)$ or $(x, y) = (3, 5)$ is the solution to the system, which is the same as the substitution method results.

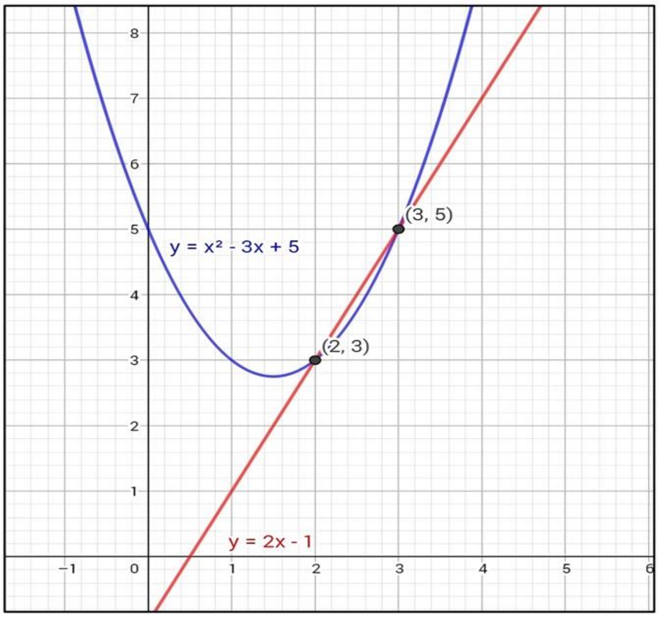


Figure 3. Graphs of $y = 2x - 1$ and $y = x^2 - 3x + 5$ (Source: Authors' own elaboration)

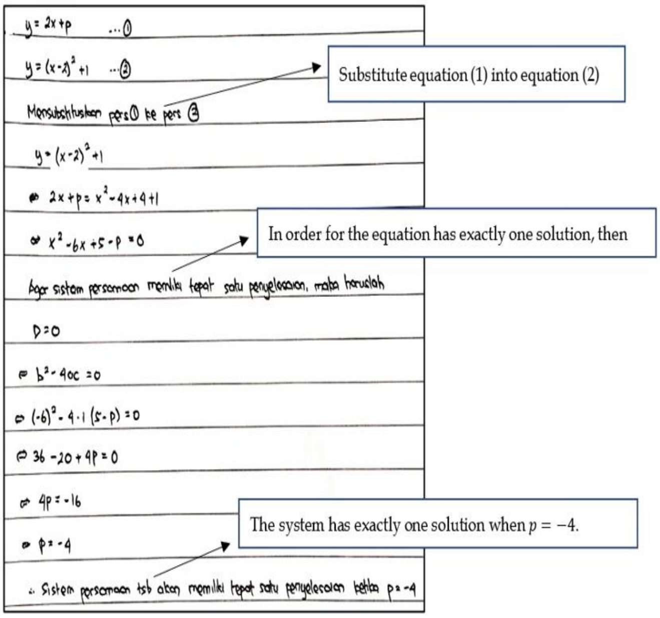


Figure 4. An example of written student work for task E2.2 (Source: Authors' own elaboration)

Table 3. Exercise tasks on systems of non-linear equations

Exercise 2. Solve each task below!
E2.1. Find solutions for each system of equations below. (a) $\begin{cases} y - x + 1 = 0 \\ y = (x-1)(x-4) \end{cases}$ (b) $\begin{cases} x - y - 1 = 0 \\ 2xy + y^2 = 5y + 6 \end{cases}$ (c) $\begin{cases} 2x + 3y - 8 = 0 \\ 4x^2 - 12xy + 9y^2 = 16 \end{cases}$
E2.2 Find the value of p in order for the system of equations below has exactly one solution. $\begin{cases} y = 2x + p \\ y = (x-2)^2 + 1 \end{cases}$.

The system of non-linear equations in two variables for the case of two equations consisting of quadratic functions was then addressed similarly to the system of non-linear equations of the previous one, i.e., from explaining the general form of the system to providing an example, discussing the solution of the systems using substitution and graph methods, and giving an exercise. **Table 3** presents the exercise (exercise 2) provided by the lecturer for his students.

One of the tasks in exercise 2 that was addressed during the classroom discussion is task E2.2. This task asks students to determine a parameter value such that the system of equations has exactly one solution. To do this, a student should be familiar with some concepts in the topic of quadratic equations. **Figure 4** presents an example of written student work for task E2.2. After the classroom discussion, the lecturer closed the teaching session by summarizing essential points concerning the topic of systems of equations. The points include the meaning or definition of a system of equations; the methods of solving systems of equations; the ways of representing systems of equations; and the applications of the concepts of systems of equations in mathematics itself, other subjects, and daily life.

Analysis of Written Work and Interviews on Solving Systems of Equations

Table 4 shows the results from pre-service mathematics teachers' written work on solving systems of equations. For the case of systems of linear equations, we found that more than 90% participants solved the tasks correctly. For the case of systems of non-linear equations, however, we found that the number of correct solutions includes 51.4%, 68.6%, and 17.2%. These findings suggest that the participants in this study have sufficient conceptual understanding and skills in dealing with systems of linear equations, but relatively lacked the case of systems of non-linear equations. Another important finding concerns the less frequent use of symbol sense strategies than procedural strategies in the solution processes, except for the case of task 3. This might indicate that the pre-service teachers have acquired more procedural skills than conceptual understanding for the case of systems of equations.

To explain the above quantitative results, we describe further how symbol sense and procedural strategies are used and are interpreted from a symbol sense perspective. In this case, we address and interpret pre-service teachers' written work and corresponding interview findings for task 2 and task 4.

First we address the findings for the case of task 2, i.e., *if* $\begin{cases} 3x + 2y + z = 7 \\ x + 3y + 2z = 3 \\ 2x + y + 3z = 2 \end{cases}$, *then find* $x + y + z = \cdots$. A typical procedural strategy for solving task 2, which is frequently found in written student work (77.1%), is carried out by applying the combination of elimination

Table 4. Findings of data analysis from written formative test on systems of equations (n = 35)

Tasks	#Correct solution		#Incorrect solution	
	#Symbol sense strategy (%)	#Procedural strategy (%)	#Symbol sense strategy (%)	#Procedural strategy (%)
1. If the system of equations has the solution $(x, y) = (1, 3)$, then $a - b = \cdots$ $\begin{cases} ax + by = 11 \\ 7x - by = 1 \end{cases}$	11 (31.4)	23 (65.7)	0(0.0)	1(2.9)
2. Consider the system of equations below. Find $x + y + z = \cdots$ $\begin{cases} 3x + 2y + z = 7 \\ x + 3y + 2z = 3 \\ 2x + y + 3z = 2 \end{cases}$	5 (14.3)	27 (77.1)	1 (2.9)	2 (5.7)
3. Consider system of equations below. Find $x + y = \cdots$ $\begin{cases} \frac{4}{x} + \frac{1}{y} = p \\ \frac{5}{x} - \frac{2}{y} = q \end{cases}$	14 (40.0)	4 (11.4)	7 (20.0)	10 (28.6)
4. Find the solution to the system of equations below. $\begin{cases} y = 2 - x \\ y = (x - 2)(x + 3) \end{cases}$	4 (11.4)	20 (57.2)	4 (11.4)	7 (20.0)
5. Consider the system of equations below. Find $2x - y + z = \cdots$ $\begin{cases} x + y + xy = 39 \\ x + z + xz = 29 \\ y + z + yz = 47 \end{cases}$	5 (14.3)	1 (2.9)	3 (8.6)	26 (74.2)

and substitution methods. For example, by eliminating the variable x from the first and second equations we obtain $7y + 5z = 2$; and from the second and third equations we obtain $5y + z = 4$. Next, by applying the elimination and substitution methods to these equations we will obtain $y = 1$ and $z = -1$. Finally, by substituting these two variables' values into one of the equations within the system, we obtain $x = 2$. Therefore, we have $x + y + z = 2$. We consider that the use of the combination method concerns more on the use of procedural skills in solving algebra tasks. Therefore, from the symbol sense perspective, the use of procedural skills can be interpreted as the skill to manipulate symbolic expressions and to check for the symbol meanings in the implementation of a procedure. In the interview, we found that even if the use of the combination method is considered a procedural strategy, students use it meaningfully to solve the task efficiently. One of the students mentioned that

> "… for solving this task, first I have to decide what methods can I use to find the values of x, y, and z. If I use the substitution method or elimination method only, then it will be lengthy. Therefore, I use the combination method, it is more efficient."

This suggests that the use of the combination method shows not only a mastery of procedural skills but also an acquisition of conceptual understanding.

A typical symbol sense strategy for solving task 2 is carried out, as follows. As the task asks to find the value of $x + y + z$, then by perceiving the whole system of equations, we can add all three equations in the system to obtain $6x + 6y + 6z = 12$. Therefore, by dividing both sides by 6, we obtain $x + y + z = 2$. This typical symbol sense strategy was confirmed in the interview. According to one of the interviewees who used the symbol sense strategy, she mentioned,

> "… when solving this task, I read and see the system comprehensively and meaningfully to see any relationships. When I see it, I add up the three equations in the system, and divide the result by 6 to get $x + y + z = 2$."

From the symbol sense perspective, the use of this strategy shows the ability to recognize symbolic relationships and to verify the symbol meanings in the process of solving a problem. **Figure 5** shows typical examples of written student work for task 2. Part a in **Figure 5**, the left part, shows the use of symbol sense strategy; and part b in **Figure 5**, the right part, presents the use of procedural strategy for solving task 2.

Next, we address the findings for the case of task 4, i.e., *find the solution to the system of equations* $\begin{cases} y = 2 - x \\ y = (x - 2)(x + 3) \end{cases}$. A typical procedural strategy for solving task 4 is carried out, as follows. Because it is known $y = y$, then $(x - 2)(x + 3) = 2 - x$. By expanding the left-hand side of the equation, we have $x^2 + x - 6 = 2 - x$, which implies $x^2 + 2x - 8 = 0$. Next, by solving this equation, we obtain $x = -4$ or $x = 2$. Finally, by substituting these values into one of the equations within the system, we obtain $y = 6$ or $y = 0$. Therefore, the solution of the system is $(x, y) = (-4, 6)$ or $(x, y) =$

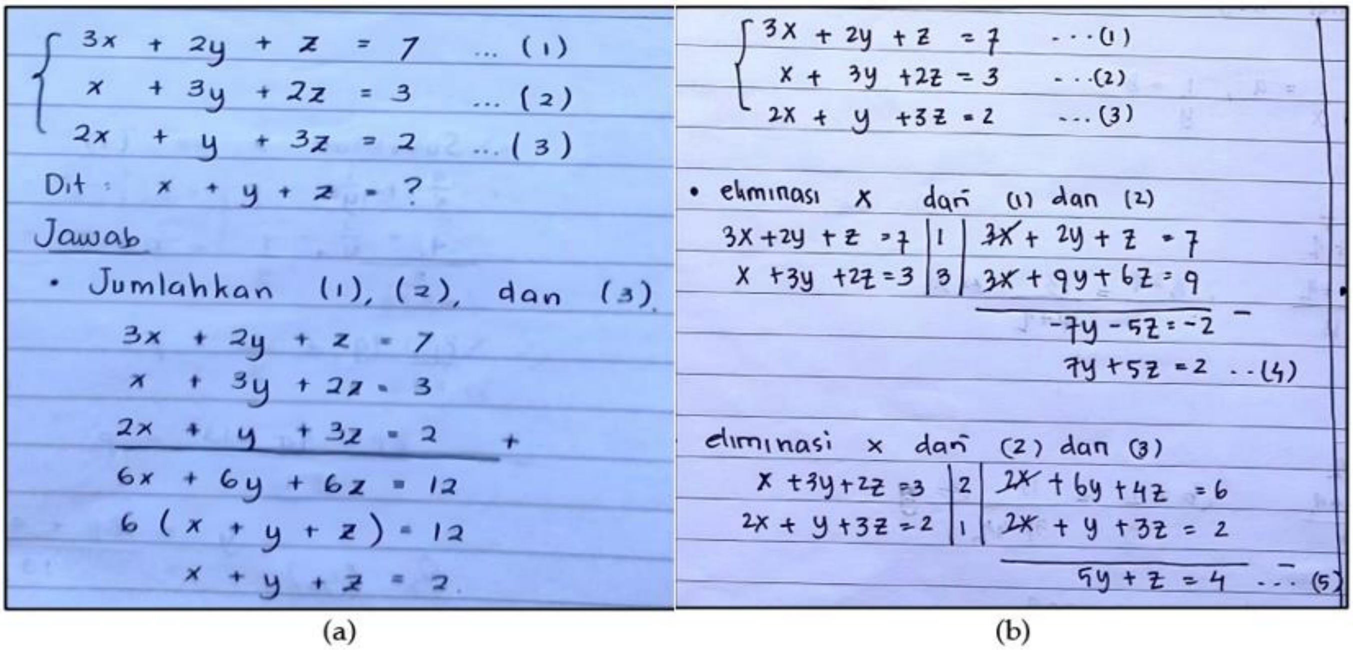


Figure 5. Examples of written work for solving task 2 using different strategies (Source: Authors' own elaboration)

$(2, 0)$. This procedural strategy is applied by using the substitution method for solving the system of equations. According to four interviewees, the use of the substitution method is clear from the given system of equations because $y = y$. In addition, one of the interviewees mentioned that

> "I do not know other methods to solve this system. I mean that I know the substitution and graph methods only. But for the graph method, for many situations I need a tool, like GeoGebra!"

As this solution process needs the skill to read and manipulate mathematical expressions, and the skill to implement solution procedure, therefore, from a symbol sense perspective, this can be considered as the procedural strategy.

A representative symbol sense strategy for solving task 4 is carried out, as follows. Because it is known $y = y$, then $(x - 2)(x + 3) = 2 - x$ or $(x - 2)(x + 3) = -(x - 2)$. Next, this equation can be written as $(x - 2)(x + 3) + (x - 2) = 0$. By applying the distributive property of multiplication over addition, we obtain $(x - 2)\{(x + 3) + 1\} = 0$ or $(x - 2)(x + 4) = 0$. From this last equation, we obtain that the solution of the system is $(x, y) = (-4, 6)$ or $(x, y) = (2, 0)$. In the interview, when one of the interviewees was asked how to solve this system of equations, she said that

> "First, I have to understand the system of equations as a whole. Next, when using the substitution method, there is one common term, namely $(x - 2)$, so I use the distributive property. In the end, when I found the solution, I check it through the substitution method to equations in the system."

This interview suggests that the participant needs the ability to read through and recognize relationships between mathematical expressions, such as recognizing the term $(x - 2)$ in the equation $(x - 2)(x + 3) = 2 - x$, during the solution process. Therefore, from a symbol sense perspective, this solution process can be categorized into a symbol sense strategy. Figure 6 shows typical examples of written student work for task 4. Part a in Figure 6, the left part, shows the use of the symbol sense strategy; and part b in Figure 6, the right part, shows the use of procedural strategy for solving task 4.

In addition to the above findings, we also noted some student difficulties in dealing with systems of equations. The difficulties include calculating and manipulating algebraic expressions that involve variables and parameters correctly, seeing relationships within a system of equations meaningfully, and forgetting to check the solution to the system. From the perspective of symbol sense, we view that the skill to do calculation and manipulation of algebraic expressions concerns the procedural strategy; and the ability to comprehend and find relationships within a system of equations for applying a more efficient solution strategy concerns the symbol sense strategy. From the interview, we found that even if the six interviewees acknowledged that checking the solution to the system was carried out, still some of the other students did not do this checking process.

DISCUSSION

Discussion on the Teaching and Learning Processes for the Case of Systems of Equations

Based on the description of results in the preceding section, we keep three points for the case of teaching and

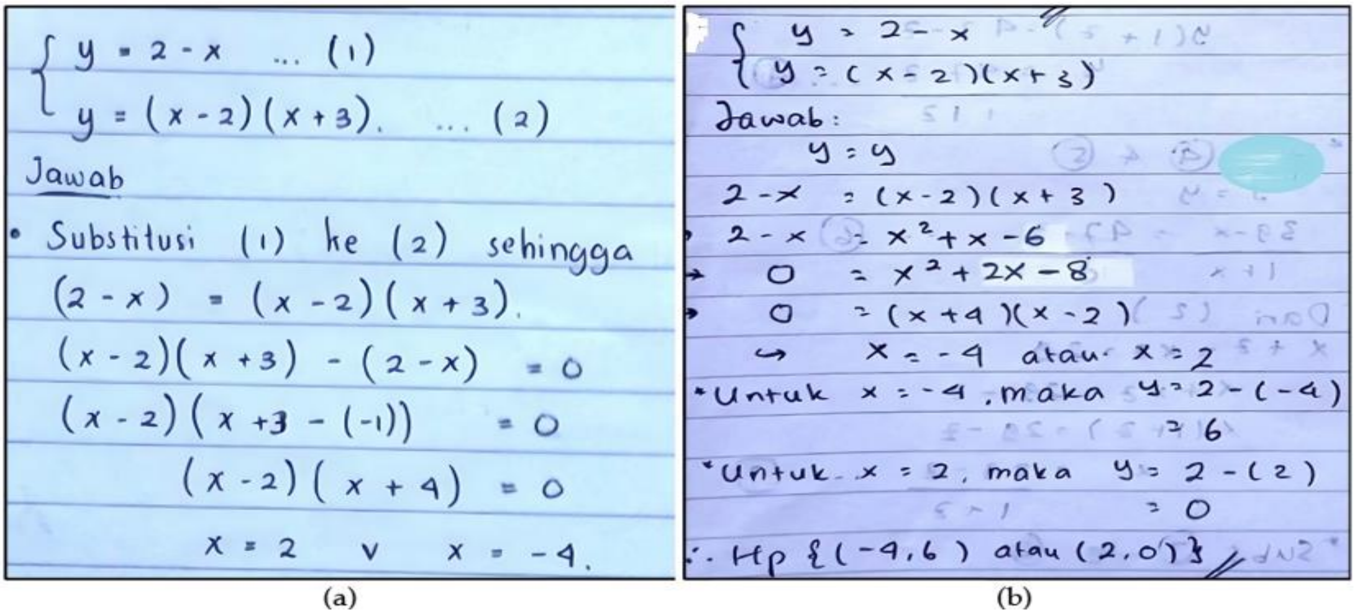


Figure 6. Examples of written work for solving task 4 using different strategies (Source: Authors' own elaboration)

learning processes. First, in general, the sequence of the teaching and learning processes for pre-service teachers for the case of systems of equations proceeds from addressing the definition and general forms of systems of equations to providing and explaining example problems, giving exercises, conducting classroom discussion, guiding conclusions, and administering an individual written formative assessment. Taking this sequence of teaching into consideration, which starts from more general ideas of the meaning of systems of equations to more specific ideas of example problems and application, we consider that the lecturer applied the deductive teaching approach (Prince & Felder, 2006; Wardani & Kusuma, 2020). By using a question-and-answer strategy, the lecturer involved students actively from one step to the next steps. From this condition, we view that even if the deductive approach was used, which is classified as one of the teacher-centered approaches (Ramsden, 1987; Stephan, 2020), the students were still actively supported to participate in the teaching and learning processes.

Second, we perceive that the content for the topic of systems of equations covered in the teaching and learning process includes definitions, representations of the systems of equations, solution methods, and applications of the topic in mathematics itself, other subjects, and daily life. However, the interpretation of a system of equations is not explicitly addressed in the teaching process. The interpretation of a system of equations includes possibilities of the number of solutions, symbolic and geometric interpretations, domain and restrictions of solutions, and possibilities of systems of equations whether having more equations than unknowns or vice versa (Proulx et al., 2009). For future research and teaching, we suggest that this interpretation probably can be addressed explicitly in the teaching process to develop pre-service teachers' conceptual understanding and skills regarding systems of equations.

Third, in the learning and teaching process, we found that the lecturer used GeoGebra as a tool for drawing graphs. This was used to explain the graph method for solving systems of equations in two variables and to visualize the idea of the solution of the systems of equations geometrically. This means that the GeoGebra is used not only as a tool for solving problems but also as an environment for improving conceptual understanding (Drijvers, 2015). Furthermore, the use of the tool seems to influence students' thinking as revealed in the interview, but it is regretted that the use of a digital tool was not allowed during the written test. Probably, in the future, the use of digital tools should be encouraged not only in the teaching and learning process but also for written assessment (e.g., Drijvers, 2018).

Discussion on the Written Work and Interview Findings for the Case of Systems of Equations

Based on the description of the written work and interview findings, we keep two points to discuss. First, in general, we observed that the procedural strategy was used more frequently than the symbol sense strategy for solving systems of equations. However, we found that the procedural strategy is used by implementing more efficient methods of solving systems of equations, whether to use substitution, elimination, or the combination of the two methods depending on the system of equations at hand. This finding, from a symbol sense perspective, suggests that pre-service mathematics teachers even if working procedurally in the solution processes, comprehend the systems of equations before executing solution methods and check meaningfully after the solutions are found (Arcavi, 1994; 2005; Kop et al., 2020; 2021). Concerning the use of the symbol sense strategy, even if only less frequently than the procedural strategy, we observed that the pre-service teachers could see, for instance, common factors and relationships between symbols in the systems of equations. From a symbol sense perspective, this suggests that the pre-service teachers can read through and get the meaning of symbolic expressions as a whole and see symbolic relationships for choosing and executing more efficient solution strategies (Arcavi, 1994; 2005; Kop et al., 2020; 2021).

Second, we found that about half the number of pre-service teachers encountered difficulties in dealing with systems of non-linear equations. The difficulties include manipulating algebraic expressions that involve variables and parameters, seeing relationships within a system of equations meaningfully, and forgetting to check the solution to the system. This finding suggests that for future teaching and research, pre-service teachers need more comprehensive treatments for understanding systems of non-linear equations. The comprehensive treatments may include providing more opportunities in the learning process, providing more exercise and classroom discussion in dealing with the topic, and designing more appropriate teaching materials on this topic for the pre-service teachers (e.g., Wilson & McChesney, 2018). In this way, it can be expected that the difficulties can be reduced and probably be avoided completely.

CONCLUSION

Based on the elucidation of the findings and discussion in previous sections, we infer the following conclusions. We concluded that the teaching and learning processes for the case of systems of equations in the SM course for pre-service teachers dominantly use the deductive teaching and learning approach. In the implementation of this teaching process, the lecturer used a question-and-answer strategy to proceed with his lesson and enriched with the use of GeoGebra software. The teaching and learning processes proceed from more general ideas of the definition of a system of equations to more specific ideas of giving examples, explaining solutions to the example problems, guiding classroom discussion, and drawing classroom conclusions. In our view, even if the deductive approach has been implemented quite well, the written test and interview findings showed that procedural skills seem to be acquired more than symbol sense ability in dealing with systems of equations. Taking this into consideration, we suggest investigating the use of teaching and learning approaches that provide more opportunities for pre-service teachers to think profoundly in comprehending and solving systems of equations. This can be implemented, for instance, by offering learning activities that explicitly ask students to use more efficient strategies and methods in solving systems of equations. Therefore, the use of well-prepared teaching approaches that have investigative characteristics seems appropriate to be explored in further research.

From the written test and interviews, this study revealed that the symbol sense strategy is used less frequently than the procedural strategy for solving systems of equations. Both strategies can be interpreted from a symbol sense perspective, as follows. The procedural strategy mainly uses the skill to manipulate symbolic expressions and the skill to verify the meaningfulness of the implementation of solution methods. The symbol sense strategy uses the ability to recognize relationships between algebraic expressions, read through the meaningfulness of a system of equations as a whole, realize the role of a symbol as a variable or parameter, and understand geometrical interpretations of a system of equations. The selection for using the procedural and symbol sense strategies depends on the relationships between equations within the system at hand. The consideration of selecting a more efficient solution strategy, in view of the perspective of symbol sense, shows the ability to read through and get the meaning of mathematical expressions before the problem-solving process. For the practical implementation of mathematics teaching in the future, we consider that the symbol sense ability can be developed through not only the topics of bare systems of equations, but also the applications of systems of equations in mathematics, other subjects, or daily life. For the balance between conceptual understanding and procedural skills acquisition of pre-service mathematics teachers, we suggest that the teaching and learning process should focus on the development of algebraic ability that ranges from basic skills such as procedural work with a local focus and algebraic manipulation to strategic work, which requires a global focus, conceptual understanding, algebraic reasoning, and creativity. We contend that the symbol sense perspective offers insight into what occurred in pre-service mathematics teachers' minds, including in the interpretations of solution strategies, comprehending the selection of solution strategies, and interpreting difficulties faced during the solution processes. In this way, the teaching and learning processes can be improved to develop pre-service teachers' procedural skills and conceptual understanding in a balanced manner.

Notwithstanding the conclusions above, we noted that the current study has several limitations. First, as this study relies on limited data of observations of the teaching and learning processes for pre-service mathematics teachers in Indonesia, we are aware that the findings of this study are not representative. However, we expect that the results can provide a portrait and insight into how mathematics teacher education is implemented in the Indonesian situation. Second, as the observations in this study were conducted from one group of mathematics education students only, we admit that the results could not be generalized. For further study, we recommend doing more extensive observations that involve more than one group of students and using relevant research methods to obtain generalizations.

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Ethical statement: The authors stated that the study, which involved 35 pre-service mathematics teachers (mathematics education students), has been conducted according to the academic ethics that adhere to the rule of the *Universitas Pendidikan Indonesia* on 16 June 2023 (Approval number: 601/UN40.4.4B/PL/2023). Written informed consents were obtained from the participants.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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