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Student engagement, conceptual-understanding, and problem-solving ability in learning plane geometry through an integrated instructional approach

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Abstract

The study investigated the impact of the integrated instructional approach on plane geometry learning using a mixed research method and quasi-experimental design on the three purposefully selected secondary schools. The researcher collected data using pre-post-tests, engagement Likert scale, observation, and interview and analyzed using ANCOVA, paired sample t-tests, and regression analysis. The intervention resulted in significant pre-post-test mean differences in experimental groups but not in the control group and showed significant differences between groups' post-intervention results. Experimental group I showed an improvement compared to the two groups. After controlling the pre-test, the integrated approach contributed 13.1%, 14.8%, and 28.24% of the variability on concept understanding, problem-solving ability, and engagement, respectively. Student engagement and conceptual understanding jointly predict problem-solving ability in all groups, with the model explaining 63.1%, 54.4%, and 38% of the variance in Experimental group I, Experimental Group II, the control group, respectively. The researcher concluded the integrated instruction significantly improved plane geometry learning and recommended its application for teachers and other stakeholders.

Keywords: problem solving ability, engagement, conceptual understanding, plane geometry, Van Hiele group guided instructional approach, group guided discovery instructional approach

INTRODUCTION

Mathematics significantly influences the development of thinking abilities, including systematic, logical, and critical problem-solving abilities (Ramadhania et al., 2022). The National Council of Teachers of Mathematics (NCTM) identifies five standard mathematical abilities among students. These standards are problem-solving, logical thinking, connection, effective communication, and representation (NCTM, 2000). To apply information, develop new understanding, and participate in critical thinking, as well as to promote conceptual understanding, the council highlights the importance of problem-solving abilities in mathematics education. Problem-solving ability is a mental process that involves finding, shaping, and reaching a final goal in any situation where there is an opportunity to improve (Kumar, 2020). It requires data interpretation, planning, outcome verification, and experimentation with different solutions (NCTM, 2000). In addition, conceptual understanding refers to the understanding of fundamental mathematical concepts, relationships, and operations, as well as the interconnected relationships that give meaning to mathematical processes (Findell et al., 2001). It enhances retention, promotes fluency, and facilitates learning contents that helps to solve real problems.

To solve real-world problems, it is crucial to integrate distinct mathematical components, each of which has special advantages. Because of its deep human links and the fact that its concepts are essential for helping students handle challenges in their everyday lives, geometry is one of the core mathematical components taught in school (Sinclair et al., 2016). Geometry instruction enhances students' visual imagination,

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Contribution to literature

- The researcher showed a practical teaching strategy to help secondary school students overcome difficulties due to a lack of background knowledge despite the lack of experimental research on this topic.
- The study investigates secondary school students' engagement and conceptual understanding in learning plane geometry, focusing on how they apply this knowledge to solve home and societal problems through this integrated instructional approach.
- According to the research, secondary school students enhanced synthesis and critically examine to solve problems by developing a more global viewpoint.

deductive reasoning, logical argument, and proof skills, enabling them to understand geometric shapes, principles, and relationships (NCTM, 2000). It is also a crucial part of mathematics that helps students understand more difficult topics in science, technology, and engineering by examining and evaluating the physical world (Zhang, 2017). Because of its strong human connection, geometry helps students solve problems in their daily lives (Jablonski & Ludwig, 2023; Singh & Kumar, 2022).

Maamin et al. (2021) emphasized how much student engagement in class activities affects their learning. Engaged students show attentiveness, active participation, and interest in study, leading to higher grades compared to disengaged students who exhibit boredom, passiveness, and low motivation (Bear et al., 2018). Researchers labeled student engagement as cognitive, behavioral, affective, and social engagements (Wang et al., 2016). Cognitively engaged students are dedicated to understanding content and focusing on tasks, while behavioral engagement involves active participation in instructions, including focus, interest, effort, task completion, and polite behavior. Affective engagement refers to an individual's attitudes, feelings, responses to academic content, and willingness to participate in their interactions with classmates and teachers. Social engagement refers to the regular interaction between students and their peers regarding academic tasks. Students must understand plane geometry to effectively tackle everyday classroom issues, as it is crucial for human understanding and problem-solving. Thus, studying geometry requires students to actively participate, which necessitates timely work completion, focused attention, intelligent academic responses, and idea exchange.

Statement of the Problem

Despite the importance of plane geometry, study results show that students usually struggle to understand and apply geometric concepts (Mirna, 2018). The school's geometry learning has not yet reached the necessary abilities (Widada et al., 2019; Yudianto et al., 2018). Students often lack an understanding of geometric concepts (Silmi Juman et al., 2022). Students start high school without attaining geometric thinking level I and level II (Alex & Mammen, 2016). Ethiopian secondary and higher education students are not interested in learning mathematics, and they struggle with problem-solving, classroom engagement, and overall performance (Walde, 2019; Walelign, 2014). Students face difficulties in mastering geometric concepts due to a lack of motivation, a perception of geometry's insignificance, and inadequate reasoning skills (Atnafu, 2016; Mirna, 2018).

The Ethiopian Ministry of Education is actively working to improve education quality at all levels, focusing on the availability of trained teachers and other essential inputs (Ministry of Education, 2009). However, the quality of mathematics education, as assessed through national learning assessments, still posed challenges in conceptual understanding and problem-solving abilities (Bethell, 2016). Students' insufficient conceptual understanding and problem-solving ability in mathematics classes are attributed to lack of confidence, poor classroom engagement, and inadequate background knowledge (Brezavšček et al., 2020; Jetu, 2019). The education sector development program V&VI in Ethiopia indicates that 10th and 12th grade students' mathematics problem-solving performance was poor and below the standard (Ministry of Education, 2010, 2020). In Ethiopia, students' participation in mathematics learning is decreasing, and they are facing difficulties in problem-solving (Belay et al., 2017; Ministry of Education, 2018). Researchers expressed dissatisfaction with the students' capacity in understanding mathematical concepts, procedures, and problem-solving techniques and highlighted the need for improved education (Mengistie et al., 2020; Shishigu, 2018).

International studies show a student-centered teaching approach to instruction greatly improves students' understanding of mathematical concepts (Nurbavliyev et al., 2022; Silmi Juman et al., 2022). However, most Ethiopian secondary school teachers employ teacher-centered teaching strategies, which are insufficient in promoting student engagement (Begna, 2017; Egne, 2022). Teacher-centered instruction can lead to students becoming frustrated, lacking self-confidence, and becoming passive consumers of knowledge (Niyukuri et al., 2020). Further, studies indicate that students face difficulties in understanding geometry due to inadequate teaching methods that do not align with their current understanding (Saadati & Celis, 2023; Sulfiyah et al., 2020). The researcher, with extensive experience in teaching mathematics from elementary to college, observed that mathematics teachers utilize a teacher-centered teaching approach.

In Ethiopian mathematics education, the teacher-centered teaching method is mostly to blame for students' poor conceptual understanding, disinterest in geometry classes, and difficulty solving problems. Consequently, a variety of factors influence the author's intention to implement an intervention. The study aimed to evaluate the effect of the Van Hiele group-guided discovery instructional approach on students' conceptual understanding, problem-solving abilities, and engagement in learning plane geometry.

Theoretical Framework

The Van Hiele Group Guided Discovery Instructional Approach (VHGGDIA) is a modified active teaching style utilizing Van Hiele, discovery learning, and positive social interdependence learning theories.

Van Hiele theory

The Van Hiele theory, introduced in the 1950s, is a geometry learning framework consisting of five phases of instruction, five levels of thinking, and five features of successful completion (Fuys et al., 1984). Howse and Howse (2015) recommend that students should complete each level of thinking in order. The visual level focuses on identifying geometric shapes based on their outward appearance rather than their characteristics.

Analytical level: Students began analysing geometrical content but were still unaware of definitions and the relationship between figures.

Informal deduction level: Students are enhancing their understanding of forms, definitions, connections, geometric figures, and logical maps and defending their findings at the informal deduction level.

Formal deduction level: Students are proficient in using induction to manage implications and can independently create proofs.

Rigor Level: Without physical models, students can explore differences and compare systems using different axioms (Rahim, 2014; Vojkuvkova, 2012).

Additionally, Van Hiele discovered common characteristics across all geometric thought levels that teachers should consider when selecting their teaching strategies.

A fixed sequence: Move through the phases in the correct order.

Advancement: The content and delivery techniques of training have a greater influence on level-to-level development than age.

Distinction: Language symbols and links vary by level.

Intrinsic- extrinsic: Inherited elements from one level becoming research topics at the next level.

Mismatch: A teacher's language and instruction hinder a student's learning growth if they are not suitable for their current level (Vojkuvkova, 2012).

Thirdly, Van Hiele developed a teaching method that employs appropriate characters to enhance students' critical thinking skills through five phases.

1. Inquiry: The teacher establishes context, helps students grasp the subject matter, ascertains their past knowledge, and directs future inquiry via dialogue and questions.
2. Guided orientation: The teacher encourages students to investigate the topic and complete brief activities while using sequenced assignments to help them understand new ideas and look at latent linkages.
3. Explanation: Students create a technical message, learn new terms, and express their thoughts on the connections they found in the second stage.
4. Free orientation: Students independently tackle challenging tasks, developing knowledge of content connections and problem-solving ability.
5. Integration: Students review, summarize, and retain learning, while the teacher provides guidance, materials, and support, using appropriate technical language and clear relational structure (Machisi & Feza, 2021; Vojkuvkova, 2012).

Van Hiele suggests that a student's mastery of a previous geometric thinking level influences their understanding of the next, and progress is accelerated through appropriate instructional methods (Vojkuvkova, 2012).

Discovery learning theory

Discovery learning theory, developed by Jerome Bruner in the 1960s, aims to improve learners' capacity to acquire new knowledge through hands-on experiences (Bruner, 1960). Discovery learning involves students creating new knowledge, assessing prior knowledge, blending new and old knowledge, and questioning, fostering risk-taking, problem-solving abilities, and unique experiences (Bicknell-Holmes & Seth Hoffman, 2000). By saving student's time and giving prompt feedback, teachers' support during exploration activities improves learning (Prasad, 2011). In guided discovery, Achara and his team developed a five-stage classroom instructional approach.

1. Simulation: The teacher simulates students, improves recall, and makes connections between new and existing knowledge by using opening questions.

2. Exploration: The teachers let the students work on their own. The teacher guides the group toward a more fruitful way by listening, offering encouragement, and posing questions.
3. Presentation: Students share with the class how they have solved the challenge. Group members had to present and debate their responses for each task.
4. Warm-up: After presentation, the teacher leads a warm-up exercise to confirm their understanding of the content, expand on ideas, and develop generalized notions.
5. Evaluation: The teacher assesses students' conceptual understanding using a teacher-made test, reviews their activity, and evaluates their output (Achera et al., 2015).

Social interdependence theory

Social interdependence refers to the interconnectedness of group members, where the actions of one member significantly influence the outcomes of the group (Johnson, 2003). Positive (cooperation) and negative (competition) are the two main categories of social interdependence. Positive interdependence is a mindset where individuals believe their success depends on the achievement of their colleagues' goals, fostering mutual support and collaboration. Negative interdependence occurs when individuals believe their goals can only be achieved if their competitively linked counterparts fail to achieve their goals, hindering each other's efforts. The social interdependence theory explains how self-interest evolves into shared interest in cooperative settings, leading to the creation of new objectives (Johnson & Johnson, 2008). **Figure 1** shows the combination of both theoretical and conceptual framework of the study.

The Integrated Instructional Approach

The Integrated Instructional Approach means the Van Hiele Group Guided Discovery Instructional Approach, abbreviated as VHGGDIA. It is a novel teaching method that incorporates the Van Hiele Instructional Model, guided discovery instructional model, and cooperative learning approach. Starting instruction with the students' existing knowledge, focusing on student participation to learn new content under the teacher's guidance, having them share what they have learned with their peers, and holding a class discussion to create a common understanding are important components of this approach. The five stages of the VHGGDIA framework were created by combining the phases of the guided discovery learning with Van Hiele's phases of teaching approach. Initially, to assess the students' current understanding, the teacher establishes goals and gives tasks based on prerequisite knowledge. In the actual stage, students learn through group discussions, share knowledge, summarize presentations, collaborate to wrap up activities, and receive teacher evaluations. **Figure 2** illustrates the stages of VHGGDIA and GGDIA in the classroom instructional process.

Brainstorming Activity: (Think-Pair-Discuss-share)

Van Hiele emphasized the importance of teachers evaluating their students' prior knowledge when introducing new content. Thus, to rehearse their prior knowledge, the instruction method involves students thinking independently, discussing their opinions with peers, debating predefined ideas in small groups, and reporting the results to the class.

Introduction: Subsequently, the lecturer divides the class into small home groups, breaks the new topic down into subtopics, and assigns them to each group member using the lottery approach. The instructor may assign a subtopic to two group members. The teacher gives a specific task to each home group member to ensure that everyone has a solid understanding of the assigned subtopics.

Discussion: The teacher encourages students to form additional expert groups by bringing together members of home groups assigned to the same subtopics. The expert group then carefully explores the subtopics by

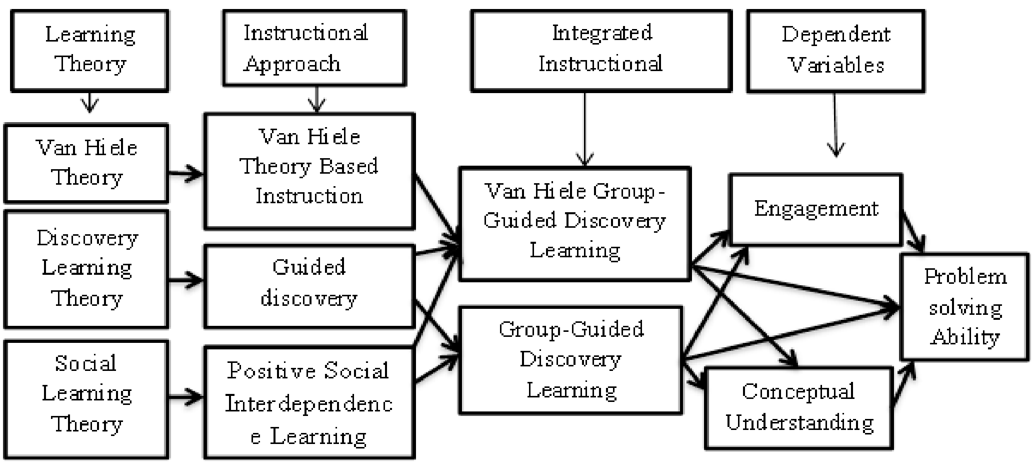

Figure 1. The theoretical and conceptual framework of the study (Bruner, 1960; Fuys et al., 1984; Johnsen, 2003)

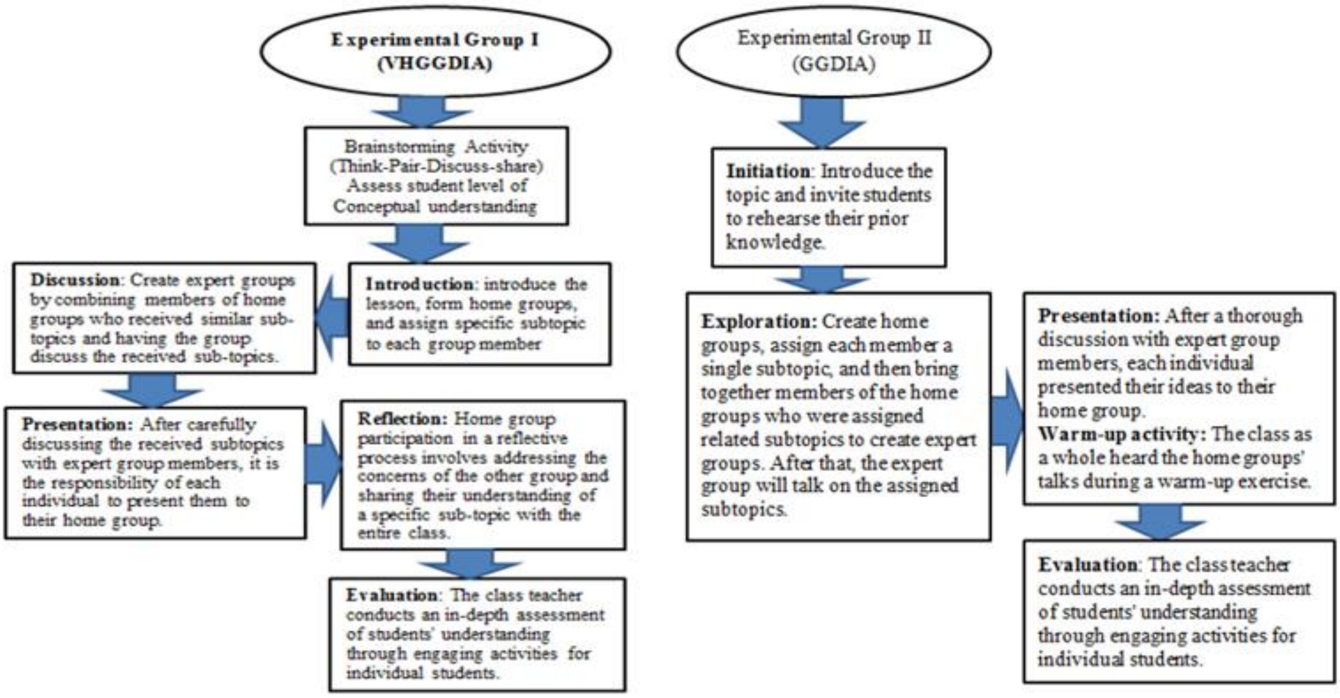

Figure 2. The framework of VHGGDIA and GGDIA (Achera et al.,2015; Vojkuvkova, 2012)

looking through various instructional resources and allocating sufficient time.

Presentation: The expert group members are responsible for carefully studying and understanding subtopics, and each member is then tasked with presenting them to their home group members. Home group members actively participate by summarizing ideas, understanding each other's perspectives, and sharing their discussions through detailed communication.

Reflection: Home groups engage in a reflective process by discussing a specific subtopic with the entire class, addressing concerns, and sharing knowledge. Finally, the teacher summarizes the conversation, and each group explains its points while monitoring participation and correcting misunderstandings at all levels.

Evaluation: The teacher conducts detailed assessments of each student's understanding using required tasks and creative assignments, evaluating each student individually.

LITERATURE REVIEW

Different instructional methods significantly influence students' conceptual understanding, problem-solving abilities, and enthusiasm for learning mathematics (Alrajeh & Shindel, 2020; Lee & Paul, 2023). The Van Hiele teaching model improves procedural knowledge and conceptual comprehension (Adeniji & Baker, 2022). Moreover, students at various levels of understanding concepts show diverse degrees of problem-solving skills, with higher-level students exhibiting more sophisticated methods (Andira et al., 2022). Thus, the Van Hiele teaching model has been found to enhance students' understanding of geometric concepts.

Mohammed and Zakariyya (2023) conducted a study using a quasi-experimental pre-post-test design with one experimental and one control group to examine how the Van Hiele instructional model affected the academic performance and geometry anxiety of secondary school students studying the topic of circles. The study revealed that students who were taught using the Van Hiele instructional approach performed better than those taught using traditional techniques. Since comprehensive geometry covers a broad field that goes beyond circles, its conclusion has limits.

To see how the Van Hiele teaching method affected students' spatial skills in platonic solids, Pujawan et al. (2020) used a quasi-experimental post-test only design. Using eighth-grade junior high school students in the Seririt sub-district, the study found that children who received instruction using the Van Hiele instructional model fared better on spatial examinations than those who received traditional teaching methods. The study did not compare pre- and post-intervention results within each group.

Furthermore, Santos et al. (2022) conducted a study on the Van Hiele phase-based learning model in teaching mathematics to eighth-grade students at Cagting High School in Bohol, Philippines. The results demonstrated that both groups had established a significant mean gain difference from pre-test to post-test. The experimental group that taught using the Van Hiele instructional model performed better than the control group. The Van Hiele instructional model is widely recommended for improving the teaching and learning of plane geometry worldwide (Machisi & Feza, 2021; Suglo et al., 2024; Yalley et al., 2021).

Similarly, research demonstrates that guided discovery learning enhances students' engagement, problem-solving abilities, and conceptual understanding and reduces rote-learning behavior in geometric concepts when compared to conventional teaching methods (Maarif & Soebagyo, 2024). This strategy helps students understanding of geometry concepts more deeply with organized activities and inspires them with presenting exercises (Maarif & Soebagyo, 2024); improves students' ability to solve mathematical problems (Yusuf et al., 2023); enhances performance in problem understanding, planning to solve problems, and strategy implementation (Rahmawati et al., 2020); and increases student problem-solving abilities as evidenced by positive attitudes among students (Yusuf et al., 2023). Cooperative group work in class discussion also positively influences student conceptual understanding through exchanging ideas (Johnson, 2003). Cooperative learning, which prioritizes direct connection and positive interdependence, is also successful in promoting rapid and simple understanding of mathematical ideas (Tamara et al., 2020). Studies show that each of these teaching approaches independently has a major influence on students' learning. However, the researcher could not discover any proof that combining these three active teaching strategies improves student learning.

Rational of the Study

First: The teacher-centered teaching approach in classrooms has little promise in enhancing student engagement and academic performance (Niyukuri et al., 2020). Furthermore, the current traditional teaching method, which results in poor student interest in mathematics (Egne, 2022) will generate a crisis in science-based instruction. The current situation may hinder the development of science and technology-literate citizens who possess critical thinking and problem-solving abilities. As a math instructor, the researchers have also seen this problem in their work life. The absence of intervention research at our study locations led me to conduct this study.

Second, international studies show that Van Hiele theory-based instruction and guided discovery learning approaches significantly improve mathematics instruction (Maarif & Soebagyo, 2024; Suglo et al., 2024). However, no actual study demonstrates how combining these three interrelated teaching strategies affects students' conceptual understanding, problem-solving abilities, and engagement in learning geometry. Thus, the study investigates the impact of the Van Hiele group-guided discovery instructional approach on secondary school students' problem-solving abilities, conceptual understanding, and engagement in learning plane geometry.

Third, the necessity of effective teaching strategies to improve students' comprehension of plane geometry is increasing. For instance, a problem-based learning strategy enhances students' problem-solving skills (Binri & Hidayati, 2022; Suminar et al., 2024); a dynamic learning environment increases student engagement (Sariyasa, 2017); the integration of GeoGebra-assisted learning and contextualized teaching strategies has been proven to enhance students' conceptual understanding (Gurmu et al., 2024; Saputra et al., 2022). However, previous studies have not explored the potential benefits of a single teaching approach in enhancing student engagement, conceptual understanding, and problem-solving abilities in geometry learning. Therefore, the study proves VHGGDIA outperforms traditional teaching methods in enhancing students' engagement, conceptual understanding, and problem-solving skills in plane geometry. This could improve the body of existing literature.

Research Objective and Research Questions

This study examines the effect of VHGGDIA on tenth-grade students in learning plane geometry to improve their engagement, concept understanding, and problem-solving ability. Specifically, the researcher designed the following research questions to provide the purpose:

RQ1 Is there a significant mean difference in students' conceptual understanding, engagement, and problem-solving ability in learning plane geometry within a group and between the groups after the intervention?

RQ2 What is the contribution of a student's engagement and conceptual understanding to problem solving ability in learning plane geometry in the three groups?

RQ3 What are the challenges of implementing VHGGDIA in learning plane geometry?

RESEARCH METHOD AND DESIGN

To understand students' experiences and latent behavior in plane geometry learning with VHGGDIA, the study used a mixed research strategy that included quantitative and qualitative methodologies (Creswell & Creswell, 2018). The study investigated the impact of VHGGDIA on students' problem-solving abilities, conceptual understanding, and engagement. The study employed a quasi-experimental, non-equivalent control group pre-post-test design in an intact classroom to avoid random assignment due to academic timetables in governmental schools (White & Sabarwal, 2014).

Data Collection Instruments

The study used a modified 5-point Likert-type scale engagement questionnaire to gauge students' engagement in learning plane geometry (Fredricks & Paris, 2016; Wang et al., 2016). The responses to positive questions ranged from strongly disagreed (1) to strongly agreed (5), while the opposite was true for negative questions. The tools were initially developed in English but later translated into the students' native language to facilitate participant interaction. Two English language experts independently translated the same document, and a third expert confirmed that the translated materials were identical.

The study utilized two-tier, close-ended, multiple-choice questions with ten items each to assess students' conceptual understanding in pre-test and post-test formats. **Table 1** details four response patterns used to manually grade student responses to tests before and after the intervention. Each item took an average of three minutes to complete, and each result was converted into hundreds for analysis purposes. Students who can list facts, identify unknowns, and suggest the best course of action will earn one point for reasoning.

The study utilized six open-ended questions for each pre-test and post-test to assess students' problem-solving abilities developed by the investigator, with each question worth five points. The study converted the total value to 100 for analysis. The pre- and post-tests are similar but not identical, focusing on logical reasoning for calculating routine and non-routine tasks with varying difficulty levels. The average time to finish each item is six minutes. The study utilized **Table 2**'s rubrics to manage student problem-solving responses before and after the intervention, following Polya's four-step strategy (Shirali, 2014).

The qualitative part of the study utilizes observation and interviews to assess the effectiveness of the intervention and identify any challenges it may face. Subban and Round's (2015) approach served as the basis for the study's observation checklist. Interview: The study conducted interviews with three teachers to assess students' understanding, participation, and challenges post-intervention using five guided open-ended questions.

The following steps outline the process of confirming the validity and reliability of the devices. First, the test designer conducted extensive literature investigations to develop the items. Second, the exam questions' relevance and suitability for conceptual understanding and problem-solving abilities were assessed by my advisor and secondary school teachers at the relevant grade levels. In response to reviewers' feedback, the exams were adjusted to match the students' academic level and language proficiency. My advisor and experienced college psychology instructor reviewed the modified engagement rating scale questionnaire, interview, and observation questions.

Moreover, the study rated the students' subjective exam scores using the Angel Group Support Center in Florida. Those with scores between 0.00 and 0.49 were given a D, those with scores between 0.50 and 1.99 a C, those with scores between 2.00 and 2.99 a B, and those with scores between 3.00 and 5.00 an A (Khan, 2015). To calculate the item difficulty index and discriminating power, divided the students into high-ability (top 27%) and low-ability (bottom 27%) groups. For problem-solving, "1" was substituted for an A grade and "0" for the others. Moreover, the multiple-choice item used '1' for correct response and '0' for the incorrect answer.

Table 1. Two-tier, close-ended, multiple-choice items response pattern

Pattern	Responses	Mark	Conclusion
00	Incorrect answer with incorrect reason	0	Lowest level of understanding
01	Incorrect answer with correct reason	1	Suggest guessing and low level of understanding
10	Correct answer with a wrong reason	2	Higher level of understanding
11	Correct answer and correct reason	3	Highest level of understanding

Source: Zhou et al., 2021; Xiao et al., 2018

Table 2. Problem solving item scoring Rubrics

Phase	Checklist (expectations)	Mark	Cumulative Decision
		0	Do not meet any expectations (0)
Understanding the problem	Restate the problem in their own words Identify asked and given information	1	Meets only a few expectations (1)
Developing the plan	Organize the problem using model and diagram. Select appropriate strategy to solve the problem	1	Meets an average expectations (2)
Executing the plan	Compute the problem Support and justify ideas were raised	2	Meets majority expectations (4)
Reflecting on the result	Verify their answer and interpret the solution	1	Meets all the expectations (5)

Table 3. Reliability coefficient of engagement rating scale

Type of questionnaires	Reliability Coefficients					
	Pilot study			Main study		
	N	Pre-test	Post-test	N	Pre-test	Post-test
Engagement	63	.911	-	166	.804	.885
Conceptual Understanding	63	.839	.822	166	.843	.819

Table 4. Item analysis of pre-conceptual understanding and problem-solving test items

Variables	Items	Upper Group	Lower Group	IDI		IDP		Decision
				Value	Interpretation	Value	Interpretation	
Conceptual understanding test items	Item #1	41	16	0.63	Moderate	0.56	Very Good	Retain
	Item #2	41	13	0.60	Moderate	0.62	Very Good	Retain
	Item #3	42	7	0.54	Moderate	0.78	Very Good	Retain
	Item #4	42	8	0.56	Moderate	0.76	Very Good	Retain
	Item #5	36	5	0.46	Moderate	0.69	Very Good	Retain
	Item #6	35	5	0.44	Moderate	0.67	Very Good	Retain
	Item #7	38	13	0.57	Moderate	0.56	Very Good	Retain
	Item #8	42	15	0.63	Moderate	0.60	Very Good	Retain
	Item #9	37	13	0.56	Moderate	0.53	Very Good	Retain
	Item #10	36	7	0.48	Moderate	0.64	Very Good	Retain
Problem solving test items	Item #1	33	0	0.37	Moderate	0.70	Very Good	Retain
	Item #2	28	0	0.31	Moderate	0.62	Very Good	Retain
	Item #3	26	0	0.29	Difficult	0.58	Very Good	Retain
	Item #4	25	0	0.28	Difficult	0.56	Very Good	Retain
	Item #5	29	0	0.32	Moderate	0.64	Very Good	Retain
	Item #6	25	0	0.28	Difficult	0.56	Very Good	Retain

Note. IDI: Item difficulty index & IDP: Item discriminating power

Table 3 displays the Cronbach's alpha reliability coefficient for the engagement rating scale questionnaire in both pilot and main studies. In addition, by giving a score of 1 for high-level understanding and 0 for low-level understanding, the KR-20 algorithm ensures the internal consistency of multiple-choice questions (see **Table 1**). Through a comprehensive item analysis, **Table 4** also further comfirmed the suitability of the two-tier multiple-choice and problem solving ability test items, the study was credible due to its group participation, data collection from various sources, prioritizing participant responses, and consultation with peers and academics (Taherdoost, 2022). The study assessed the information's suitability for secondary school students, identifying its limitations, suggesting further research, and emphasizing its diverse usage. To guarantee transferability, advisors and academics were given the opportunity to further debate the emergent subject and analytical techniques.

Pre-Intervention Activities

Population and sampling techniques

The study conducted in Ethiopia Sidama region, Hawassa city administration, focusing on tenth-grade students in government secondary schools during 2022/2023. Several factors led the study to select this location, including budgetary limitations, a lack of empirical studies on mathematics education in the city, and as a math teacher, the researcher's personal observations of students' poor academic performance and lack of enthusiasm for math classes. Furthermore, this grade level was chosen for the study because, in compliance with the nation's national education policy, a mathematics textbook for grade 10 should thoroughly comprehend the concepts of plane geometry at this level and motivate students to learn how to apply them.

Base line data collection

Initially: The study was approved by the mathematics department of Bahir Dar University in Ethiopia. The approval letter was sent to the secondary school administrators at the research location.

Second: The study surveyed eleven secondary schools in the city, focusing on their infrastructure (classroom settings, internet access, and a well-organized library) and stakeholder willingness to implement an intervention, with four schools selected based on survey results.

Third: A pilot study was conducted on the fourth school, which has 63 students, located far from the other three chosen schools.

Fourth: Three math teachers from three schools were chosen based on their willingness, qualifications, and experience, and three groups were randomly selected from their previous teaching groups. The three teachers provided written consent. The study intentionally selected the two nearest schools for intervention and the farthest school for control to ensure continuous monitoring and minimize data contamination. Two nearby schools were randomly assigned to Experimental Group I and Experimental II to deliver VHGGDIA and GGDIA, respectively. Moreover, the farthest school implemented traditional teaching methods.

Fifth: Teacher training for experimental group I took place on the VHGGDIA stages, whereas teacher training for experimental group II took place on the GGDIA stages. Both teachers introduced a new teaching strategy on trigonometric functions during regular sessions for a week before incorporating it into the intervention.

Sixth: The pre-intervention data was collected using an engagement rating scale questionnaire and pre-tests for conceptual understanding and problem-solving ability. The study involved 166 students, with 44.64% male and 55.36% female, 56 from experimental group I, 54 from experimental group II, and 56 from the control group.

During the intervention period

Following the pre-intervention data collection, the actual intervention period lasted for six weeks. Using material from a tenth-grade student mathematics textbook, the intervention focused on plane geometry (theorems on triangles, special quadrilaterals, more on circles, and regular polygons) (Ministry of Education, 2010). The instruction began with group discussions on triangle theorems in the first week. The second week's first three days focused on activities related to theorems on triangles. Students discussed special quadrilaterals for the final two days of the second week. The third week was devoted to discussing special quadrilaterals. Students worked on circle-related activities during the fourth week, which carried over into the first three days of the fifth week. During the final two days, they talked about polygons. In the sixth week, student project work served as a summary for the discussion, and the teacher concluded by summarizing the entire chapter.

Post-Intervention Data Collection procedures

The study administered a pre-intervention engagement rating scale questionnaire for the second time and teacher interviews to gather information about the student progress and received data for the challenges of the intervention in learning plane geometry during the seventh and eighth weeks. Likewise, the study assessed students' conceptual understanding and problem-solving abilities using post-test items. The experimental group I oversaw closely adhered to the VHGGDIA five phases that were described in the introduction session. The experimental group II teacher followed Achara and his colleagues' phases for GGDIA implementation, while the control group teacher continued conventionally.

ANALYSIS

The study analysed student engagement, conceptual understanding, and problem-solving abilities across groups using pre-test, post-test, observations, and interview to discover issues.

Students' Engagement, Conceptual Understanding, and Problem-Solving Ability

For the first research question, **RQ1**, the study utilized statistical tests such as ANCOVA, paired sample T-test, and regression analysis to compare groups, with initial assumptions outlined.

1) The observations are independent as the data scores of the individuals are consistent across different schools.
2) Levene's test demonstrates that the variances of results of the variables are uniform across all groups ($p > .05$).
3) Since the skewness and kurtosis values in each of the three groups range from -1 to 1, the dependent variables are normally distributed (Demir, 2022).
4) The researcher utilized Levene's test and ensured homogeneity of variances in the dependent variables by controlling the covariates.

Analysis of engagement, conceptual understanding, and problem-solving ability between groups

The study initially collected data from three groups and assessed the impact of the VHGGDIA intervention after reducing confounding variables. Like time (giving equal time to all groups) and teacher influence (assigning teachers with similar experience, the same qualification, and similar involvement in teaching mathematics). Supportive instructional material (All three groups used similar instructional materials), topic coverage (All groups covered the same topics), and controlled pre-test effect using ANCOVA.

Table 5 shows pre-test mean score results for experimental Group I, experimental Group II, and the control group. First, conceptual understanding was 43.93, 42.15, and 42.14, respectively. Second, problem-solving abilities were 29.00, 27.90, and 26.34, respectively. Third, engagement in learning plane geometry was 3.39, 3.49, and 3.65, respectively. The study indicates that all groups exhibit comparable behaviors across all three variables.

Table 6 displays the adjusted post-test mean scores for experimental group I, experimental group II, and the control group after controlling the covariates pre-test. First, conceptual understanding was 67.09, 54.38, and 44.94, respectively. Second, problem-solving ability was 40.13, 33.57, and 27.12, respectively. Third, student engagement was 4.19, 3.88, and 3.66, respectively. The finding shows that experimental group I outperforms as compared to the other groups. This suggests that the

Table 5. Descriptive statistics

Variables	Mean								
	Experimental group I			Experimental group II			Control group		
	N	M	SD	N	M	SD	N	M	SD
Pre-test CU	56	43.93	24.40	54	42.15	26.06	56	42.14	26.92
Pre-problem solving	56	29.00	16.158	54	27.90	16.156	56	26.34	14.083
Pre-Engagement	56	3.39	0.247	54	3.49	0.349	56	3.65	0.320

CU=Conceptual understanding, PSA=Problem solving ability, M=mean, SD=standard deviation

Table 6. CU, PSA, and engagement after intervention pre-test as a covariate

Variables	Groups	N	Unadjusted		Adjusted	
			Mean	Std. Deviation	Mean	Std. Error
Post-conceptual understanding	Experimental Group I	56	67.32	20.67	67.09	3.18
	Experimental Group II	54	54.26	25.59	54.38	3.24
	Control Group	56	44.82	26.08	44.94	3.18
Post-problem solving	Experimental Group I	56	40.48	13.48	40.13	1.729
	Experimental Group II	54	33.61	13.93	33.57	1.759
	Control group	56	26.73	13.352	27.12	1.730
Post-engagement	Experimental Group I	56	4.19	0.37	4.19	0.047
	Experimental Group II	54	3.88	0.38	3.88	0.047
	Control Group	56	3.66	0.28	3.66	0.047

Table 7. The ANCOVA result on Engagement, PSA, and CU

Variables	Sources	df	Mean square	F	Sig.	Partial Eta squared
Post-conceptual understanding	Pretest	1	3941.126	6.964	0.009	0.041
	Groups	2	6911.392	12.213	0.000	0.131
	Error	162	565.899			
Post-problem solving	Pretest	1	3013.038	18.033	0.000	0.100
	Groups	2	2359.626	14.122	0.000	0.148
	Error	162	172.588			
Post-engagement	Pretest	1	0.057	0.480	0.489	0.003
	Groups	2	3.782	32.138	0.000	0.284
	Error	162	0.118			

Table 8. Tukey HSD test comparing groups on post-test result

Variables	Group (I)	Group (J)	MD (I-J)	Sig.	95% confidence interval	
					Lower bound	Upper bound
Post-conceptual understanding	Exp. I	Exp. II	13.06	.015	-9.55	13.11
		Control group	22.50	.000	-9.44	13.02
	Exp. II	Control group	9.44	.105	-11.33	11.34
Post-problem-solving ability	Exp. I	Exp. II	6.86	.024	0.74	12.99
		Control group	13.75	.000	7.68	19.82
	Exp. II	Control group	6.89	.023	0.76	13.01
Post-engagement	Exp. I	Exp. II	0.311	.000	0.18	0.441
		Control group	0.546	.000	0.41	0.681
	Exp. II	Control group	0.235	.001	0.10	0.367

VHGGDIA is more effective than the GGDIA and traditional teaching methods in helping students acquire plane geometry.

Further, Table 7 presented the significant difference between the groups in each variable. As First, conceptual understanding was ($F(2, 162) = 12.213$, $p < .001$, $\eta^2 = .131$) with the partial eta-squared being 0.131. Second, problem-solving ability was ($F(2, 162) = 14.122$, $p < .001$, $\eta^2 = .148$) with the partial eta-squared being 0.148. Third, student engagement ($F(2, 162) = 32.138$, $p < .001$, $\eta^2 = .284$) with the partial eta-squared is 0.148. According to Cohen, all values fall into the small effect size (Cohen, 1988). The result indicated that after controlling the covariates, the independent variables contributed 13.1% to conceptual understanding, 14.8% to problem-solving ability, and 28.4% to engagement of the variability in learning plane geometry.

Student conceptual understanding in Table 8 revealed a significant mean difference between Experimental Group I and Experimental Group II ($p < .05$, $d = .563$) and between Experimental Group I and the

Table 9. Paired sample T-test on engagement, CU, and PSA

Variable	Group	N	MD	SD	SE	t	df	Sig.
(Post – pre) Conceptual Understanding	Experimental Group I	56	23.39	30.05	4.02	5.826	55	.000
	Experimental Group II	54	12.11	37.64	5.12	2.365	53	.022
	Control Group	56	2.68	32.10	4.29	.624	55	.535
(Post – pre) Problem-solving	Experimental Group I	56	11.49	17.26	2.307	8.39	55	.000
	Experimental Group II	54	5.71	14.11	1.920	2.44	53	.004
	Control Group	56	0.39	19.32	2.582	1.36	55	.884
(Post – pre) Engagement	Experimental Group I	56	0.79	0.41	0.055	14.54	55	.000
	Experimental Group II	54	0.40	0.52	0.071	5.57	53	.000
	Control Group	56	.012	0.41	0.055	0.218	55	.828

Note: MD = Mean difference, SD = Standard deviation, SE = Standared Error

control group (p < .001, d = .963). The experimental group II students showed no significant difference in student conceptual understanding test results compared to the control group (p > .05). The student problem-solving ability showed a significant mean difference in Experimental Group I and Experimental Group II (p < .05, d = 0.494), Experimental Group I and the control group (p < .05, d = 1.181), and Experimental Group II and the control group (p < .05, d = 0.660). Similarly, the student engagement showed significant mean difference in Experimental Group I and Experimental Group II (p<.05, d = 1.086), Experimental Group I and control group (p<.05, d = 2.073), and the experimental group II and control group (p<.05, d = 0.902). The finding shows the experimental group I significantly improved student conceptual understanding, engagement, and problem-solving ability compared to the other two groups. The VHGGDIA reveals a larger than typical effect size compared to the traditional teaching method.

Analysis of engagement, concept understanding and problem-solving within a group

The study examined the shift in student engagement, problem-solving ability, and conceptual understanding from pre- to post-test in all three groups.

Table 9 shows the improvement of the three variables in experimental group I. First, post-conceptual understanding (M = 67.32, SD = 20.67) and pre-conceptual understanding (M = 43.93, SD = 24.40) improved by 23.39 points. The study shows a significant mean increase from pre- to post-conceptual understanding (t (55) = 5.83, p < .001). Second, post-problem-solving ability (M = 40.48, SD = 13.48) and pre-problem-solving ability (M = 28.99, SD = 16.16) improved by 11.49 points. The study found a significant increase from pre- to post-problem-solving ability (t (55) = 8.39, p < .001). Third, post-engagement (M = 4.19, SD = 0.36) and pre-engagement (M = 3.39, SD = 0.25) improved by 0.79 points. The study found a significant increase from pre- to post-engagement (t (55) = 14.54, p < .001).

Similarly, the experimental group II demonstrated a mean increase from pre- to post-intervention in the three dependent variables. First, post-conceptual understanding (M = 54.38, SD = 25.59) and pre-conceptual understanding (M = 42.15, SD = 26.06) improved by 12.11 points. The finding shows a significant mean increase from pre- to post-conceptual understanding (t (54) = 2.37, p < .05). Second, post-problem-solving ability (M = 33.61, SD = 13.93) and pre-problem-solving ability (M = 27.90, SD = 16.16) improved by 5.71 points. The finding shows a significant improvement from pre- to post-problem-solving ability (t (54) = 2.44, p < .004). Third, post-engagement (M = 3.88, SD = 0.38) and pre-engagement (M = 3.49, SD = 0.35) improved by 0.40 points. The result showed a significant improvement from pre- to post-engagement (t (53) = 5.57, p < .001). However, the control group students were non-significant in conceptual understanding (p = .884 > .05), problem-solving ability (p = .884 > .05), and engagement (p = 0.828 > 0.05).

Qualitative Responses

Teacher interview: The study coded the participant teachers as TT1 from Experimental group I, AT from Experimental group II, and TT2 from control group. The study categorized interview responses as: students' engagement, concept understanding, and problem-solving ability.

Student engagement in classroom discussion

"Do your students actively participate in geometry instruction? If not, why? If yes how?"

TT1: Initially, due to poor prior knowledge, students exhibited passive behavior, lack of participation, and disinterest in learning math. However, the intervention increased the student's active participation, completed tasks, explained solutions, analyzed diverse perspectives, and improved their self-confidence (TT1, 25 May 2023, 2:30).

AT: After the intervention, students' classroom participation improved significantly, and their ability to apply learned ideas greatly enhanced practical activities, and the group work allowed students to concentrate and discuss ideas and prepare them for plane geometry concepts (AT, 25 May 2023, 2:30).

TT2: During my teaching, students often exhibited passive behavior, such as not writing notes, instead of participating in the class, disrupting class presentations, and struggling to apply classroom knowledge in real-world situations. Students' insufficient prior knowledge was the primary reason for their disruption in class (TT2, 26 May 2023, 8:30)

"Do you think active practice in the classroom lesson contributed to greater engagement in plane geometry lesson compared to the same plane geometry content would traditionally taught?"

TT1: The strategy facilitated students' knowledge acquisition through collaborative work, enhancing their understanding of solving real-life problems using classroom concepts (TT1, 25 May 2023, 2:30).

AT: The intervention greatly improved students' engagement in class discussion and understanding of ideas to solve problems, but the duration was insufficient (AT, 25 May 2023, 8:30).

TT2: Due to time constraints and students limited prior knowledge, I used the lecture method, resulting in less engagement in classroom instruction and a lack of understanding of the lesson. If students have enough prior knowledge, I believe an active teaching strategy is better than the traditional method (TT2, 26 May 2023, 2:30).

Students' conceptual understanding and problem-solving ability

"How well do you believe that the students understood the concepts of plane geometry during the lessons that required physical activity?"

TT1: The intervention significantly improved student engagement in plane geometry instruction for practical application, despite most students lacking basic knowledge in primary school, which hinders their ability to connect classroom knowledge to real-world contexts (TT1, 25 May 2023, 2:30).

AT: The intervention greatly enhanced students' practical activities, and the group work allowed students to concentrate on discussing ideas and prepared them for plane geometry concepts (AT, 25 May 2023, 2:30).

TT2: My students have difficulty applying classroom knowledge in real-world situations, leading to difficulty in practical activities (TT2, 26 May 2023, 8:30).

"Do you think the teaching strategy that you applied contributed to greater concept understanding in plane geometry instruction?"

TT1: The implemented active teaching strategy-facilitated students' in-depth knowledge acquisition through collaborative discussions with peers, thereby enhancing their understanding of the subject matter as needed and increased their ability to solve problems (TT1, 25 May 2023, 8:30).

AT: The implemented active teaching method reduced students' dependence and increased their self-confidence, allowing them to reach their desired level independently, but it needs enough time (AT, 25 May 2023, 2:30).

TT2: Due to students' weak background knowledge and shortage of time, I used the lecture method, but this method is not satisfactory for students to fully understand concepts (TT2, 26 May 2023, 8:30).

Researcher Observation

The study categorized the observation results into themes such as teacher instruction, student's reaction, student concept understanding, and problem-solving ability as shown in **Table 10**.

Teacher instruction in group discussion

According to the initial observations, teachers in the two experimental groups first struggled with the new teaching approach but later adapted successfully. Throughout the intervention, the experimental Group I teacher effectively helped students through the VHGGDIA five phases by asking hypothetical questions and gave them plenty of chances to defend their answers in various ways. Additionally, if students are struggling with an activity, the teacher asks them to demonstrate how they completed it and offers advice on how to attempt a different approach. Although the instructor of experimental group II followed the GGDIA five phases in a similar manner, the students found it difficult to finish the assignment in each session, and the teacher had difficulties controlling their tardy participation because of time constraints. Using a teacher-centered approach, the teacher in the control group reviewed previous content, introduced new topics, gave notes, summarized lectures, and occasionally assigned homework. These practices continued till the end of the intervention.

Table 10. Observation protocol

Major theme	No.	Items	Observed Evidence-comments
Classroom environment	1	Attractiveness of the classroom	
	2	Classroom resources that help to teach plane geometry	
	3	Availability of instructional media in plane geometry classes	
Teacher and students contact for instructional process			
Teacher activity			
Teachers' instruction in classroom discussion	4	Giving activities on appropriate difficulty level	
	5	Inviting students to do activities in different ways	
	6	Relating the activities to day-to-day practice	
	7	Inviting students to do an activity independently by giving responsibilities and sharing their ideas in a group	
	8	Inviting students who faced the challenge in doing an activity to try other ways and ask others to receive a hint	
Student activity			
Engagement in classroom lesson	9	Tried to solve problems in different form	
	10	Resilient to negative micro-feeling and instead using them as a signal or change strategy	
	11	Tried to do an activity individually with full of confidence	
	12	Tried to communicate with others to understand concepts	
	13	Tried the problem using different techniques to get solution	
Concept understanding in classroom lesson	14	Discussing concepts with peers to solve problems	
	15	Relate learned idea to real situation to solve real problems.	
	16	Connect geometric concepts to other fields	
	17	Clearly and neatly reflect their work to whole class	
	18	Respond to other students and listen attentively to others	
Problem solving ability	19	Restate the problem in their own words	
	20	Understand the problem by identifying given and unknown facts	
	21	Considers alternatives to answer divergent solutions	
	22	Develop appropriate strategies to solve a problem	
	23	Checked their work after completing the task	

Student reaction in classroom lesson

Based on the initial observations, students in all three groups exhibited passive behavior during assignments, poor communication, and a lack of curiosity about the teaching method, which hindered practice in both individual and group settings. However, a week later, students in the two experimental groups adapted to the new teaching approach; they contributed to class discussions by completing exercises, explaining solutions, and analyzing perspectives. Experimental group I students became more comfortable asking their teacher or peers questions that they were unsure about and were more willing to try any activity in a variety of ways. Students often attempt to solve problems in various ways, rather than giving up if they don't receive the correct answers. Despite each student's efforts to complete a task independently and confidently share it with their peers, some students struggled with group discussions. Students in experimental group II not only openly discussed the topic with group members but also attempted to address it independently. Some students struggle with innovative tasks and questioning, leading to inaccurate answers and unsuccessful attempts at alternative approaches. **Figure 3** shows sample Experimental groups student's engagement in group

Figure 3. Experimental groups' discussion: Group I (left); Group II (right) (Source: Field study)

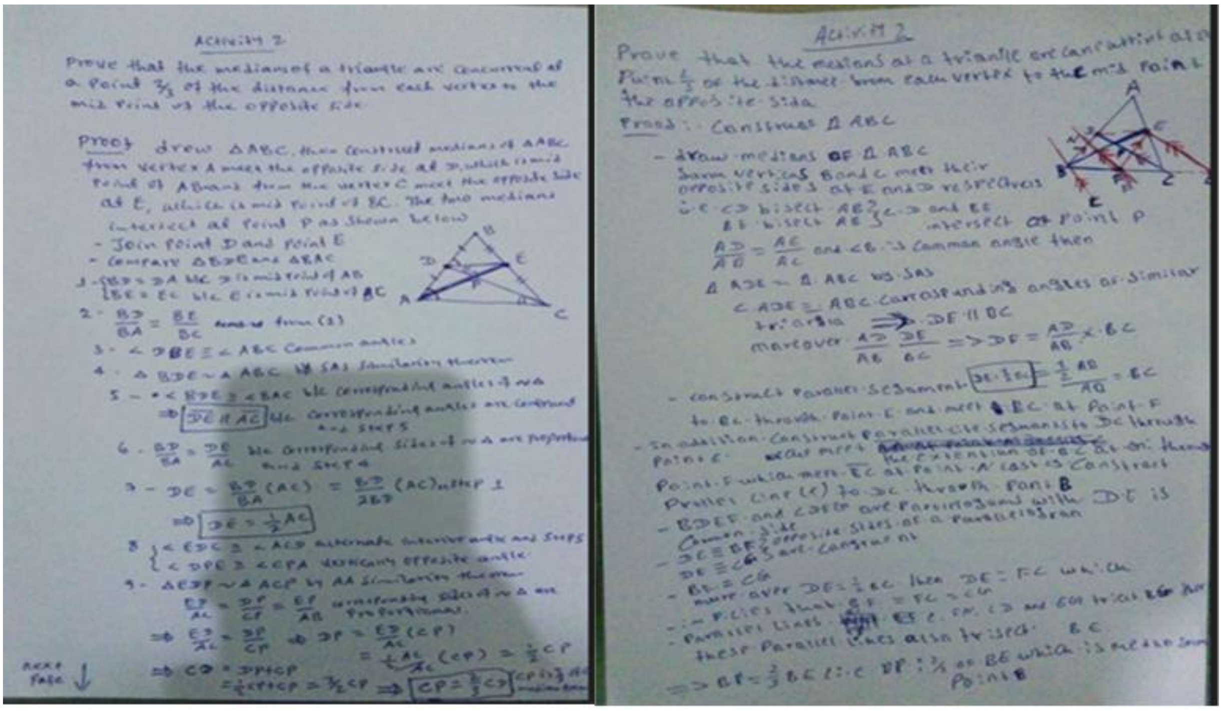

Figure 4. Experimental groups' works: Group I (left); Group II (right) (Source: Field study)

discussions. The control group students maintained face-to-face seating arrangements throughout the intervention.

The control group students remained silent in class discussions, avoiding active participation and instead focusing on the teacher's presentation. Students did not respond to the questions the teacher asked. Until the end of the intervention, the students in the control group continued to follow this pattern during the teaching-learning process.

Student conceptual understanding in classroom lesson

Initial observations showed that students in three groups have lacks enough prior knowledge, which makes it difficult for them to apply concepts, link ideas, participate in group discussions, and present their work. The experimental groups significantly enhanced their understanding of concepts through group discussions and their ability to apply them in real-world situations. By applying ideas to real-world circumstances, improving communication, making connections between new and existing information, and giving concise explanations, the experimental group I greatly increased their understanding of concepts. Similarly, the students in experimental group II shared their knowledge with the other members. While some students actively connected the contents to real-world situations to address difficulties, others found it difficult to do so since they had not thoroughly practiced the necessary information. Group members have talks, but they take longer to complete and need more help from the teacher. Since the teacher's lecture takes up most of the time, the control group won't see any interaction.

Problem solving ability in classroom lesson

At first, there were no indications that any of the group members could solve problems. But as the session progressed, the experimental group demonstrated improvement in managing both routine and non-routine tasks. Others finished the assignment by carefully examining the problem, restating the pertinent elements, and using these concepts to develop a plan, solve problems, and then validate the results, while some students struggled to understand the problems, especially word problems. Although most of the students in experimental group II struggled with these tasks, they also demonstrated some improvement in their problem-solving abilities. The control group showed significant issues in improving students' problem-solving abilities. For the question "Prove that the medians of a triangle are concurrent at a point two-thirds of the distance from each vertex to the midpoint of the opposite side." The sample group answer in the two experimental groups was displayed in **Figure 4**. The teacher proved the statement on a chalkboard to the control group, who attentively listened.

Contribution of Conceptual Understanding and Engagement on Problem Solving Ability

To answer the second research question, **RQ2**, the study found a positive correlation between conceptual understanding and engagement with a correlation coefficient < .70, indicating that the variables are not multicollinearity relations.

Table 11. Experimental group I regression analysis for PSA and component of variables

Problem-solving abilities (PSA)						
Multiple R= .795	R^2 = .631			Adj R^2 = .617		
ANOVA Table						
	Sum of Squares		df	Mean Square	F	Sig.
Regression	6304.983		2	3152.492	45.37	.000
Residual	3682.623		53	69.483		
Total	9987.606		55			
Variables in the equation						
Variables	r	B	Std. Error	β	t	Sig.
CU	.790	.486	.062	.745	7.86	.000
Engagement	.449	.175	.176	.095	0.998	.323

Table 12. Experimental group II regression analysis for PSA and component of variables

Problem-solving abilities (PSA)						
Multiple R=0.737	R^2=0.544			Adj R^2 = .526		
ANOVA Table						
	Sum of Squares		df	Mean Square	F	Sig.
Regression	5593.210		2	2796.605	30.380	.000
Residual	4694.771		51	92.054		
Total	10287.982		53			
Variables in the equation						
Variables	r	B	Std. Error	β	t	Sig.
CU	.709	.351	.054	.644	6.478	.000
Engagement	.410	.392	.184	.212	2.134	.038

Regression analysis for student problem solving ability and mediating variables

The study utilized multiple linear regression analysis to investigate the influence of engagement and conceptual understanding on problem-solving abilities. VIF values below 10% and tolerance values above 10%, indicating no multicollinearity influence between predictive variables. There is no significant correlation between the independent and residual variables (p=0.190>0.05), and the residual variable has a normal distribution. The scatter plot reveals no issues with the regression model's heteroskedasticity, and the Durbin-Watson test results show no correlation between independent and residual variables, with a range of 1.5 to 2.5.

Table 11 revealed that the combination of the two independent variables significantly predicts student problem-solving ability, with the model explaining 63.1% of the variance and F (2, 53) = 45.37, p < .001. Furthermore, the coefficients determine the impact of each factor on problem-solving ability. The results revealed that students' conceptual understanding significantly predicts their problem-solving ability (β=.745, t=7.86, p<.001). However, students' engagement had a non-significant impact on students' problem-solving ability (β = .095, t = .998, p = .323 > .05). The study shows that by increasing students' conceptual understanding, the VHGGDIA significantly enhanced their capacity for problem-solving.

Table 12 revealed that together, the predictors explained 54.4% of the variation in experimental group II students' problem-problem-solving F (2, 53) = 30.38, p < .001). This variation's substantial level indicates that students' problem-solving abilities significantly shaped by their conceptual understanding (β = .644, t = 6.478, p < .001) and engagement (β = .212, t = 2.134, p = 0.038 < 0.05) taken together. The study revealed that GGDIA significantly enhanced students' problem-solving abilities by enhancing their engagement and conceptual understanding.

Table 13 shows that 38% of the variation in problem-solving ability was jointly explained by the predictors (F (2, 53) = 16.223, p < .001). The study found that students had a significant level of conceptual understanding (β= .613, t = 5.642, p< .001), but their engagement in learning plane geometry was non-significant (β=.024, t= .223, p>0.05). The study exposed that the traditional method improved students' problem-solving ability by improving their concept understanding.

The overall quantitative data finding shows that the VHGGDIA significantly improved students' engagement, problem-solving abilities, and conceptual understanding. Similarly, the GGDIA significantly enhanced students' problem-solving abilities and conceptual understanding of plane geometry. However, the traditional teaching approach did not significantly impact students' engagement, problem-solving abilities, and conceptual understanding. Students who learned through VHGGDIA experienced greater benefits compared to those in other groups. The qualitative data analysis shows improvement in experimental groups, but control group students struggled to meet goals and

Table 13. Control group regression analysis for PSA and component of variables

Problem-solving abilities (PSA)						
Multiple R=0.616	R^2=0.380			Adj R^2 = .356		
ANOVA Table						
	Sum of Squares		df	Mean Square	F	P
Regression	3723.353		2	1861.677	16.223	.000
Residual	6081.849		53	114.752		
Total	9805.202		55			
Variables in the equation						
Variables	r	B	Std. Error	β	t	p
CU	.616	.222	.057	.613	5.648	.000
Engagement	.085	.058	.259	.024	.223	.824

had no interest in plane geometry learning. The quantitative and qualitative analyses revealed a positive correlation.

Analysis of Challenges in Implementing VHGGDIA

During the implementation period, both teachers and students faced challenges, raising the third research question, **RQ3**. The study utilized observation and teacher interviews to gather data, categorizing results into classroom situations, student reactions, and teachers' prospective.

Classroom situations

The initial section of the observation checklist revealed some classroom issues. The initial observation indicates a significant resource shortage, many students, and a lack of two-way communication, which hinders effective intervention. The method encourages active participation and introspection through unstructured group discussions, but a large group size restricts task sharing and reflection. The teaching method faces challenges due to students' lack of prior knowledge and the absence of active teacher-student interaction. The teacher's interview results also showed that numerous classroom students obstructed the intervention. For example:

> TT1: It was difficult to guide students in group discussions because there are so many students in a class.

Students' reaction

The observation result shows since the students were accustomed to more conventional teaching methods, this was a novel experience for them. The class faced challenges due to fear among students and time constraints, making it difficult to follow discussions on novel topics requiring prior knowledge. Students struggled to engage in discussions about a new subject due to their reluctance to transition from passive to active learning.

The interview responses also show students find this teaching approach difficult since they believe it calls for more difficult assignments.

> Interviewer: What challenges did you observe during the intervention period?
>
> TT1: The students' dislike of mathematics was preventing them from participating in class discussions as we had hoped. In addition, students find it difficult to finish the assigned topic in the allocated time periods since the intervention is carried out inside the mandated government school schedule (TT1, 25 May 2023, 2:30).
>
> Interviewer: Can you add additional point to what you are feeling?
>
> TT1: The method helps students come up with their own solutions; it's crucial to observe how creative and critical they are. For them to focus on their task, a quiet and convenient environment is necessary. Teaching a class of fifty or more students is challenging as a result (TT1, 25 May 2023, 2:30).
>
> TT1: The new teaching strategy requires effective communication; however, some students find it difficult to express and synthesize their experiences in both written and spoken forms. This made the new strategy's implementation challenging. (TT1, 25 May 2023, 2:30).

Teachers prospective

Preparation of the instruction and excessive amount of teacher preparation for implementation was noted by the observer. Lesson planning was a challenge for the teacher because thoughtful teachings support students in understanding the links between each step. The teacher's everyday obligations required them to prepare ahead of time for the six-week experiment.

Implementation: Teachers' follow-up was crucial for the success of the strategy, as students were disengaged and delayed task completion due to insufficient assistance. The finding indicates that the large class size poses a challenge for teachers to simultaneously address every student's issue.

> TT1: Over the past 25 years, I have embraced a teacher-centered approach, but I require adequate training to effectively implement this strategy in my classroom. The group management faced challenges due to the students who were sometimes talkative and disturbed the class. (TT1, 25 May 2023, 2:30)

The students thoroughly enjoyed the activities, despite facing numerous challenges throughout the intervention process. Most students found the model stages to be an enjoyable method for learning plane geometry. The variety of activities they engaged in was a significant factor that made it exciting. In summary, the VHGGDIA is essential for teaching geometry, and some difficulties may be avoided with careful preparation and planning.

DISCUSSION

This study investigated how VHGGDIA affected students' problem-solving abilities, conceptual understanding, and level of involvement when studying plane geometry.

Effect of the Van Hiele Group-guided Discovery Instructional Approach

Effective learning environments connect students and competencies, requiring careful design and implementation of activities aligned with teacher pedagogy to enhance their effectiveness (Kocagul, 2024). The study examined the impact of VHGGDIA on students' engagement, conceptual understanding, and problem-solving abilities in learning plane geometry. Further, the study investigated the influence of engagement and conceptual understanding on students' problem-solving abilities.

The intervention significantly improved student engagement and conceptual understanding, with experimental group I students showing a larger than typical effect size compared to the control group (see **Table 8**). The method increases students' interest in studying plane geometry, helps them understand the new concept, and eventually improves their problem-solving abilities.

This study's findings are in line with the previous findings (Silmi Juman et al., 2022). This study provides appropriate time for students to rehearse their previous information since Van Hiele states that those who lack sufficient background knowledge struggle to achieve the desired outcome.

The result shows that VHGGDIA significantly improved students' conceptual understanding, engagement, and problem-solving ability in learning plane geometry. This teaching method blends the guided discovery instructional style with the Van Hiele model. Instruction in plane geometry benefited from each of these teaching approaches. Various study results supported this research finding (Machisi & Feza, 2021; Mohammod & Zakariyya, 2023; Santos et al., 2022; Yalley et al., 2021).

What makes this study different from previous studies is that this study focuses on integrating strategies for high school students who lack background knowledge, helping them understand primary concepts and complete grade-level competencies within time. The intervention in Experimental Group II also positively impacted on student engagement, conceptual understanding, and problem-solving ability compared to the control group, but not as significantly as Experimental Group I.

In addition, the VHGGDIA showed a statistically significant change within the group, as evidenced by the comparison of pre- and post-test results. The GGDIA also showed a significant change in pre- and post-test results, but not as much as the VHGGDIA. The result of this study was consistent with research led on elementary school mathematics instruction using the Van Hiele phase-based learning approach (Santos et al., 2022).

In general, VHGGDIA significantly improves student engagement, conceptual understanding, and problem-solving abilities compared to GGDIA and conventional teaching methods. Additionally, students' participation in plane geometry lessons, conceptual comprehension, and practical problem-solving skills were all markedly improved by using VHGGDIA.

Contribution of Engagement and Concept Understanding on Problem Solving Ability

Moreover, the study found that students' problem-solving abilities were significantly enhanced by the combination of expected student engagement and conceptual understanding when trained using VHGGDIA, GGDIA, and traditional teaching methods. Students treated with VHGGDIA demonstrated improved conceptual understanding, leading to improving their problem-solving abilities. The finding was supported by previous research findings (Al-Mutawah et al., 2019; Kholid et al., 2021). The study found that student engagement in plane geometry learning did not significantly enhance their problem-solving ability compared to conceptual understanding. However, the strategy significantly increased students' engagement in plane geometry learning. The study indicates that VHGGDIA can significantly improve students' concept understanding, which is significantly more predictive of their problem-solving abilities than their level of engagement.

The result shows that the VHGGDIA was an effective strategy in attaining students' conceptual understanding to improve their problem-solving ability. For each score of student conceptual understanding, the student's problem-solving ability increased by .75. In teaching plane geometry using the VHGGDIA, Van Hiele instructional model has a considerable impact on students' conceptual understanding rather than engagement to improve their problem-solving abilities.

However, engagement has weak and non-significant influences on student problem-solving abilities. The result contradicts with the finding of Lein and his colleagues (Lein et al., 2016). From the participants' responses, one of the reasons for this problem was the large number of students in the classroom reduces students' engagement to actively participate in-group discussion. In experimental group II, students who engaged in learning plane geometry and gained a conceptual understanding had good problem solvers. Different research study results supported this finding (Al-Mutawah et al., 2019; Kholid et al., 2021). Still, the more predictive value was the conceptual understanding. The control group student conceptual understanding is a good predictor of student problem-solving abilities.

Generally, the researcher concludes that the VHGGDIA considerably enhanced student engagement, conceptual understanding, and problem-solving abilities. Additionally, by improving students' conceptual understanding, the method was more successful in improving their problem-solving skills.

Challenges of VHGGDIA in Learning Plane Geometry

Active teaching includes attractive classrooms, active engagement, enough supportive materials, effective teacher planning, and observation, significantly influence students' performance (Malik, 2018; Pajarillo-Aquino, 2019). The VHGGDIA encourages student participation and communication between students and teachers and emphasizes the significance of plane geometry concepts in solving daily problems.

First, the study shows that instructors' use of VHGGDIA in the classroom is hampered by lack of time, resources, large student populations, and inconvenient sitting arrangements. This conclusion aligns with the research that Eison discovered (Eison, 2010).

Second, some students are often unmotivated to learn mathematics, particularly plane geometry, since they perceive it as unimportant and dull. These negative attitudes about mathematics make it harder for students to participate in class discussions and express their thoughts to others. These unwanted feelings made it difficult to implement VHGGDIA in the classroom discussion.

Thirdly, because of the students' lack of prior knowledge, the teacher finds it difficult to utilize this teaching style. The approach also requires careful preparation, starting with the background information and working up to the new material that will be covered. The teacher's patience becomes exhausted during the process.

CONCLUSION

The study came up to the conclusion that the VHGGDIA is important for encouraging students' abilities to solve problems. Thus, when creating and implementing instructional activities for the VHGGDIA, teachers need to take students' developmental phases into account. Instructional activities are also appealing components that enhance students' conceptual understanding and foster participation, which in turn improves their capacity for problem-solving. The results further emphasize how crucial teacher-student contact is. Accordingly, the research suggests that both classroom and outside activities should foster effective communication between teachers and students for student success. During the intervention period, challenges like unattractive classroom environment, lack of supportive instructional materials, and time constraints hindered the successful implementation of the VHGGDIA.

The authors feel that teachers and individuals looking for additional in-depth research might greatly benefit from the study's findings. This research's primary limitations were due to school scheduling; the intervention phase lasted just six weeks, so it would be best to continue the therapy for a longer amount of time; and the study primarily focused on one city due to resource constraints. As a result, the study suggestions: The teacher should help the student overcome everyday obstacles by combining classroom instruction with real-world activities outside of the classroom. Teachers should provide a relaxed learning atmosphere for knowledge sharing and begin the class using the students' existing knowledge. This activity promotes teamwork, helps students retain prior knowledge, and gets them ready for future classes. Teachers should focus on activities that enhance students' engagement and conceptual understanding, thereby improving their problem-solving abilities.

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Declaration of interest: The authors declare that there are no conflicts of interest.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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