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Rural teachers' meanings about teaching of decimal metric system

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Abstract

This article describes and identifies the personal meanings that the multigrade schoolteacher has in the teaching of the decimal metric system, through the facets of didactic mathematical knowledge. In various investigations supported by this theory, the need to apply the notion of facets or didactic suitability as an instructional process in which teachers are knowledgeable and competent in an academic setting, has been visualized. In this sense, the research is framed in teachers of different areas of knowledge that guide mathematics in which ten teachers participated. The data collection is based on the analysis of a diagnostic test and a semi-structured interview. Finally, as one of the relevant results of the research, three epistemic configurations are obtained from the personal meanings identified in the teachers interviewed.

Keywords: teaching, decimal metric system, personal meanings, didactic mathematical knowledge, practicing teachers

INTRODUCTION

In the rural context, the role of the teacher is fundamental for the educational development of students; among the necessary factors, the teacher's didactic mathematical knowledge (DMK) is considered important, and this knowledge not only implies mastering mathematical content, but also understanding how to teach it effectively (Castro et al., 2018; Quintero-Bacca, 2022). Moreover, in rural areas, resources are mostly limited and teaching conditions vary. Therefore, the mathematical competence of the teacher becomes even more relevant, as it is highlighted that the difficulty in the application of play in the classroom is related to the attitude of both the teacher and the student. Specifically, in the rural context, teachers may face additional challenges due to lack of access to technological resources and apathy towards methodological changes. However, playfulness and innovative pedagogical strategies can be powerful tools to foster mathematical learning in these communities (Castro et al., 2023; Cossío, 2014; Grijalba et al., 2019; Hernández Barbosa, 2014; Herrera & Buitrago, 2015; Soler, 2016).

The above opens a gap to describe some of the needs of multigrade teachers (Ruíz, 2021) among which are planning, articulation between learning, mastery of mathematical knowledge, the activities proposed from the content of the academic period, and the textbooks used for the design of classes. The above also emerges due to a major problem, which is that in Colombia, one in four educators work as teachers in rural contexts (Zamora Guzmán, 2018), the vast majority of whose academic work is carried out at the preschool and primary school levels. These teachers come from various disciplines, as well as with different levels of study, which makes teaching mathematics in particular more complex. On the other hand, it is necessary that public education, essentially in the rural context, requires teachers whose teaching strategies are based on collaborative learning and joint accompaniment, which will help to reduce student dropout rates and allow the teacher to have an approach and interaction with the social, cultural and emotional context of the student.

Thus, one of the objectives in the teaching and learning process in the area of mathematics is to promote creativity in the elaboration, creation and solution of problems that allow the analysis of different didactic mathematical purposes (Burgos et al., 2024), starting from the mobilization of knowledge to the difficulties presented in the classroom by students and teachers. Therefore, the need to develop creative skills in rural teachers that allow them to teach effectively through the teaching of mathematical problems becomes stronger.

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Contribution to the literature

- The study identifies the personal meanings of ten teachers in a rural context through three epistemic configurations involving the concept of the Decimal Metric System.
- The study analyses the facets of the teacher's Mathematical Didactic Knowledge through the components and characteristics that make it up, focusing on the rural school teacher.
- The study shows that the personal meanings of teachers in rural contexts influence their conceptual construction of the affective facet.

This is how the teacher's DMK plays an important role, since it has been a scaffolding for the knowledge and competences that can be fostered in the teacher to be able to articulate with anthropological and semiotic aspects (Breda et al., 2017).

Some rural schoolteachers in teaching notions that shape the decimal metric system (DMS) such as the concept of standard length make use of both mathematical and non-mathematical language (Mwale, 2021). In the mathematical language they embrace the standard units of length, meter and centimeter, as well as the procedures for measuring and comparing lengths of objects, while in the non-mathematical language they associate the verbal expressions of width, length, without mathematical specification or rigor. Likewise, they use the contents and pedagogical practices to introduce the standard units of length and make this concept comprehensible to students in different dimensions and tasks (Arnal-Bailera & Arnal-Palacián, 2023).

Calculating length is a common everyday activity and a universal mathematical concept shared by different cultures (Gómezescobar et al., 2023). Hence the interest in discovering the strategies used by preschool and first grade students from different countries to measure length using the ruler as a didactic tool. However, instead of the conventional ruler, four specially designed rulers are used to evaluate the measurement strategies implemented by the children, even so, difficulties are perceived in the measurement process of both preschool and first grade students, which is why teachers who provide support and guidance to children during measurement activities have a positive impact on their learning.

The learning process of DMS concepts is limited in the classroom by the sole use of textbooks, since only the identification of the concept of the unit of measurement is involved; the equivalences that exist between them; carrying out conversions; ordering natural numbers and decimals; operations with these numbers; and the student's experience with the mathematical concept is left aside (Cerezo Gamarra, 2021). In this sense, Cerezo Gamarra (2021) states that it is necessary for students to acquire real measurement experiences in order to implement various strategies that allow them to learn and develop in the everyday context, such as: the construction of measurement and scale, problem-solving, experience with measurements and estimation, making use of real situations to prevent students from executing a mechanized internalization. According to the above, the teacher should not only be a provider of information, but a facilitator of learning, developing critical thinking and problem-solving skills, applying knowledge to real world situations, and the teacher can achieve this through different strategies.

Thus, in this article we are interested in investigating: What personal meanings do teachers in a rural context have in relation to DMS? For this, the theoretical lens we will use is DMK of the teacher proposed by the onto-semiotic approach (OSA) to knowledge and mathematical instruction. The methodology is mixed in which the responses of 10 teachers to a diagnostic test and an interview were analyzed.

THEORETICAL FRAMEWORK

DMK, developed by Pino-Fan and Godino (2015), offers a holistic perspective to understand the knowledge that teachers possess in the field of mathematics teaching. This approach conceptualizes teacher knowledge through three fundamental dimensions: the mathematical dimension, the didactic dimension and the meta-didactic-mathematical dimension. In particular, the didactic dimension emerges as a focal point in this research, as it proposes six facets that are crucial for the ongoing analysis. These facets are considered subcategories of mathematics teacher knowledge, reflecting the depth and breadth of specialized knowledge required in the effective teaching of mathematics.

The epistemic facet addresses the teacher's mathematical mastery, while the cognitive facet focuses on the teacher's understanding of the students' cognitive processes. On the other hand, the affective facet refers to the teacher's knowledge of the emotional and attitudinal aspects of his or her students. Likewise, the interactional facet concentrates on the dynamics of interaction that occur in the classroom, while the mediational facet considers the resources and media that can enhance students' learning. Finally, the ecological facet addresses the various contexts–social, political, economic and curricular–that influence the management of student learning. Together, these facets outline a comprehensive picture of the knowledge needed to address the challenges of mathematics education in today's educational context.

Pedagogical content knowledge emerges as the cornerstone of the teacher's background, synergistically merging the depth of mathematical content with the pedagogical strategies necessary for effective teaching (Galimova et al., 2023). From the theoretical perspective, two fundamental definitions emerge that delineate the nature of content knowledge in the field of mathematics education. First, there is the concept of common content knowledge, which is defined as "knowledge that is used in the work of teaching in ways common to how it is used in many other professions or occupations that also use mathematics" (Hill et al., 2008, p. 377). This type of knowledge is not limited exclusively to the educational domain but extends to a variety of contexts, where mathematical skills are used, such as performing basic arithmetic calculations, applying common mathematical formulae or interpreting graphs and tables.

On the other hand, there is the concept of specialized content knowledge, which refers to "mathematical knowledge and skills unique to teaching" (Hill et al., 2008, p. 400). This type of knowledge is specifically geared to the task of teaching mathematics and comprises skills that go beyond mere mathematical competence. These skills include the ability to represent mathematical ideas accurately, to facilitate the explanation of mathematical rules and procedures, and to analyze and understand unusual methods of problem-solving. For example, specialized content knowledge enables the teacher to explain abstract mathematical concepts clearly and concisely, to adapt explanations to different levels of understanding, and to detect common mathematical errors in students. Together, these two dimensions of content knowledge provide a comprehensive view of the competences needed for effective mathematics teaching.

From a semiotic perspective, this model analyses the communication of mathematical ideas through representational registers in teaching and learning, considering the importance of symbolic cognition in the construction of mental registers by both teachers and students and the meanings that emerge in the understanding of the nature of mathematical knowledge. Several theoretical perspectives apart from semiotics have examined or explored structures in the meanings that allow this understanding of the nature of mathematical knowledge, some of these perspectives are ontological and epistemological (Godino et al., 2022). In this sense, it becomes fundamental to teach and learn mathematics to develop a deep understanding of the concepts and their applications, apart from identifying what conceptions, linguistic interpretations, mental structures the teacher has to transit and guide the student to the construction of mathematical objects.

This research describes the notions of the six facets that articulate in a coherent and systematic way the model of DMK (Pino-Fan & Godino, 2015), starting from the definition of facet as the particular way in which a mathematical concept can be perceived, understood or taught by the teacher (**Figure 1**). It is a way of approaching a mathematical object from different

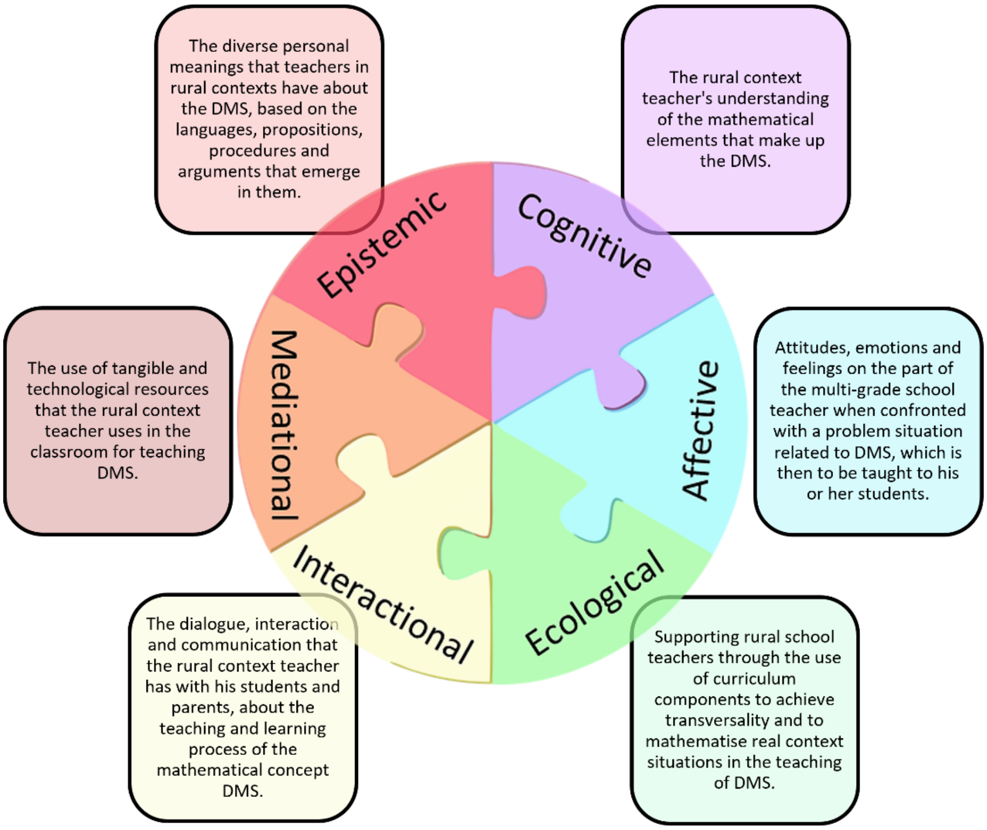

Figure 1. Definitions of the facets of the DMK model in the rural context (Source: Authors' own elaboration)

Table 1. Typologies of personal meanings of teachers in rural contexts in relation to DMS

Type of meanings		Description
Global		The teacher in a rural context identifies generalized notions of the mathematical concept DMS, without arriving at a specific, institutional definition.
Declared		This corresponds to definition expressed by the teacher in the rural context that he/she has mentally incorporated, making use of representation registers, in which, these expressed definitions can be correct or incorrect from the institutional association that emerges from mathematical object DMS.
Achieved	Initial	This meaning accounts for the pre-images in relation to the notion of DMS, in which the teacher's prior knowledge of the rural context is involved.
	Final	This meaning accounts for internalization of mathematical object DMS in rural context teacher, from relationship of previous & new knowledge, as well as conversion into different registers of representation reaching institutional definition of mathematical concept.

perspectives, recognizing its various dimensions and possible interpretations (meanings). This research shares this definition with the adaptation in the rural context, between the personal meanings of the multigrade schoolteacher and the intended institutional meanings, since when teachers have an empowerment and understanding of DMS mathematical object, they will be able to transmit it and their teaching processes will be suitable to facilitate effective learning experiences for their students.

Continuing with what was mentioned in the previous paragraph, the facets of DMK model are described, which will provide a methodological route for the research and a novel contribution in relation to the contextual framework of the study. In order to subsequently elaborate the epistemic configurations and personal meanings conceived by practicing teachers.

Therefore, one of the crucial elements in our research is the personal meanings that the teacher has mentally constituted about DMS. These meanings that each person, whether the teacher or the student, gives to a mathematical concept are generated according to their experience, previous knowledge and cultural context (Godino et al., 2022). From OSA, this type of meaning is called personal meaning. In which the cognitive and epistemic dimensions are related, so that mathematical information is processed involving connections with previous knowledge, in order to identify patterns and apply strategies to solve problems.

The above allows us to mention that personal meanings are not isolated from social, cultural and cognitive practices, but are immersed in them. This is why it is necessary to analyze the facets of how a personal meaning of a mathematical object is constructed and how it influences, specifically in teachers in the rural context, ecological, affective, didactic and mediational elements. That is, in the context of mathematics teaching, how the teacher has the ability to apply or develop characteristics of the components of the facets that allow them to possess knowledge of mathematical content, reflect on their practice to continuously improve their teaching and, ultimately, promote meaningful and effective learning in their students (Breda et al., 2017).

With regard to personal meanings in our research and in relation to the study population, the mathematical object, and theoretical references such as Font (2007), the types are described in **Table 1**. Now, personal meanings in our research are routed to the basic learning rights (BLR) and mathematical thoughts (MT) that are considered and considered fundamental to determine or analyze whether a teaching and learning process is appropriate or optimal in a specific educational context (Breda et al., 2017; Beltrán-Pellicer & Godino, 2020). That is, what attributes are being considered by the teacher in a rural context for the construction of a mathematical concept, and how this construction considers the phenomenology of the concept, the curricular organizers, the resources used for its understanding and the didactic transposition that can be implemented to achieve an encapsulation of the mathematical object.

Otherwise, the teacher in a rural context must recognize that there is an articulation between mathematical thinking and DMS, with BLR at the time of focusing the planning of their practices. **Figure 2** presents an articulation of BLR with mathematical thinking and DMS in this regard.

METHODOLOGY

The research focuses on a mixed methodology (Giberti et al., 2023), developed in three stages: first, a diagnostic test (pre-test), an analysis grid and an interview protocol related to the facets of DMK, and the mathematical object of study (DMS) were designed. Also, at this stage, a validation of the three instruments was carried out through the review of experts in the field of the rural context and mathematics education. In a second stage, the diagnostic test and the semi-structured interview were implemented in the classroom with teachers from rural educational institutions in Colombia. Finally, the third stage involved analyzing the data obtained, using the grid previously designed based on coding, frequencies and histograms, as well as the construction of the epistemic configurations that emerge in the data obtained from the practicing teachers, which allowed us to analyze their mental structure (personal meaning) associated with the mathematical content, and in a synthetic way how the teachers in the multi-grade

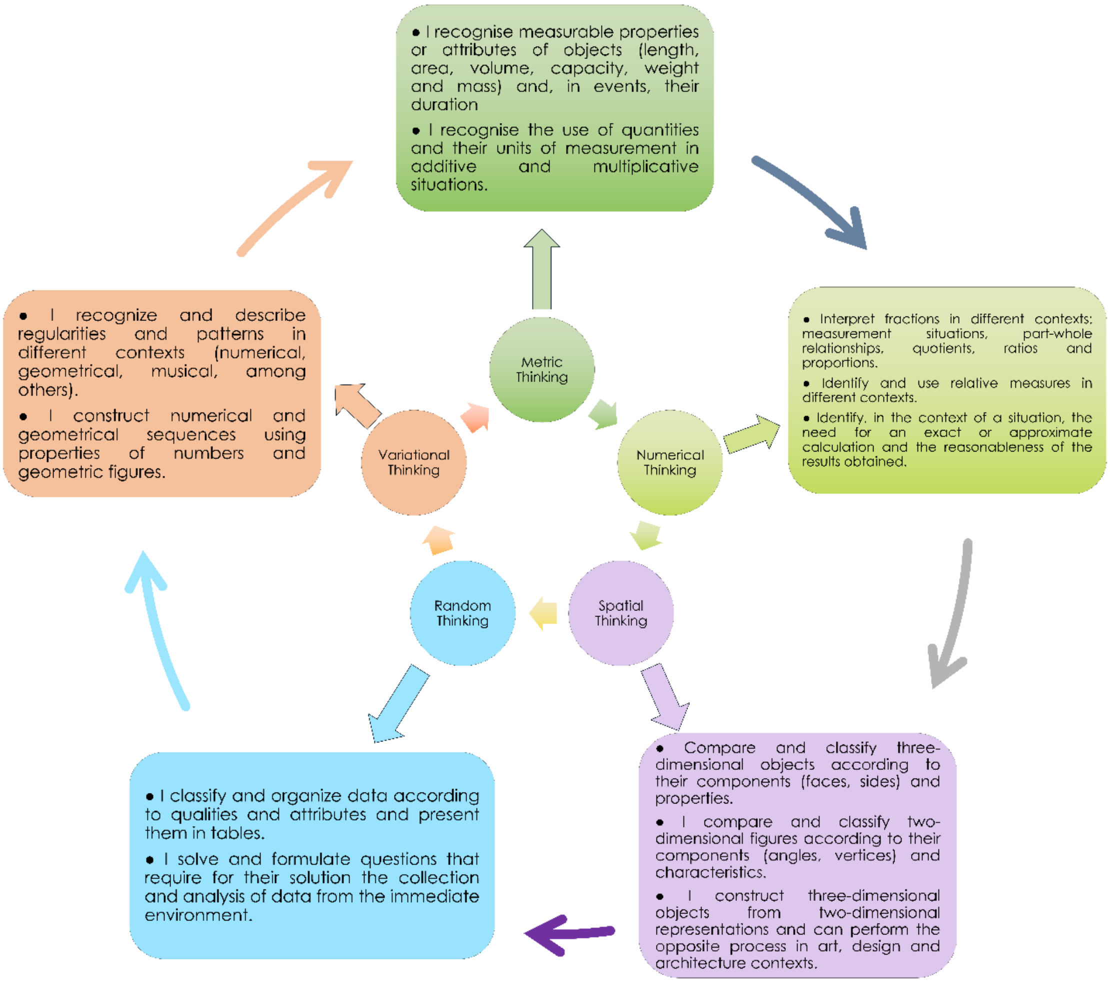

Figure 2. Articulation between BLR & MT (Source: Authors' own elaboration)

schools in the research are understanding the mathematical content for their teaching.

Description of Research Participants

For the selection of the participants, the following characteristics were considered:

(1) being a practicing mathematics teacher,

(2) belonging to a rural educational institution (IER) in the department of Quindío (Colombia),

(3) working in an IER with more than four sites, and

(4) teaching from fourth to sixth grade.

This allowed for a total of ten practicing teachers, ranging in age from 30 to 50 years old. All participants had an undergraduate degree, except for two with a master's degree and two with a diploma.

Data Collection Instruments

To analyze the information obtained in this study, three instruments were designed:

1. A diagnostic test.
2. A semi-structured interview.
3. A grid of the six facets of DMK model.

The diagnostic test (pre-test) considered the facets of DMK and an articulation between BLR of Colombia and five mathematical thinking–PMs–(numerical thinking and number systems, variational thinking and algebraic and analytical systems, random thinking and data systems, spatial thinking and geometric systems and metric thinking and metric or measurement systems) that make up the curriculum of the rural public education institutions.

This test was structured by ten items. This allowed us to elaborate the interview and the analysis grid. Subsequently, a validation by experts was carried out, with the participation of two PhDs and a master's in mathematics education. They have conducted research in the field of inclusive education, specifically in rural schools.

In particular, the semi-structured interview had the objective of investigating the teacher's conceptions of the mathematical object, his emotions, his methodological teaching strategies and his communication with his immediate environment. For this purpose, four questions were established:

Q1. Describe how you start your classes when you are going to introduce new mathematical concepts.

Table 2. Ecological facet

Component	Code	Characteristics	Frequency(%)
Adaptation to curriculum	EcF1	Knows the mathematical object (content), its implementation corresponding to the curricular guidelines of the educational institution.	30
Openness towards didactic innovation	EcF2	Integrate new technologies (calculators, computers, & TACS) into the teaching and learning process of DMS.	40
Intra- & inter-disciplinary connections	EcF3	It promotes relationship between DMS with other areas of knowledge, on intra- & inter-disciplinary contents, seeking a transversalities.	30

Q2. Do you think it is important to keep yourself updated on a daily basis on the strategies and methodologies that can be implemented in your classroom in the teaching of mathematics? why?

Q3. How often do you do this kind of updates?

Q4. How do you allocate your time to teach DMS to your students?

These questions also made it possible to verify what the teacher stated in writing during the diagnostic test. Finally, the collection instrument related to DMK facets grid began with the analysis of the questions one by one, in order to subsequently describe the indicators or characteristics that made up the six facets named in **Figure 1** and code them, in order to identify which of these facets of DMK model stood out most in the exercise of a mathematics teacher in the context of multi-grade schools (rural schools) and which epistemic configurations emerged in the development of real situations close to context in which the teacher worked.

It is important to make clear that the coding of the information obtained in the study from the qualitative approach was done in two dimensions. First, a description and code was established for the components that made up each facet, for example: the component of openness to the didactic innovation of the ecological facet, the acronym EcF was used, alluding to the facet studied (ecological facet), the letter C determining that it was a characteristic and the number one the position of the characteristic, so that it was denoted EcF1. The description was adapted to the context of the teacher, such as the rural context, to the mathematical concept of DMS and to the needs that were observed from theory and praxis in the teaching of mathematics in multi-grade schools. To then give rise to the second dimension, which consisted of analyzing which facets were most outstanding in the multi-grade school teachers at the time of the development of the diagnostic test based on these characteristics and especially which personal meanings stood out when analyzing the pre-test in which we were given elements of how the concept of DMS was internalized, constructed mentally, cognitively in the teacher and how their environment, both emotional, pedagogical and mediated, impacted on this conceptual construction. The aforementioned was symbolized in a regular heptagon that represented the epistemic configuration of the practicing teachers in the rural context.

In the case of the quantitative approach, as with the qualitative approach, the information was also coded in two dimensions, the first was to determine a frequency represented by percentages that resulted from the analysis of the traffic light of the characteristics of the six facets, which showed the total number of teachers who met the criteria. These frequencies were placed in each of the facet tables. This gave way to a second dimension, which was the elaboration of a statistical representation (bar chart) of the numerical data obtained in the analysis.

RESULTS

The analysis of the data obtained from the research began with the identification of the various characteristics of the facets stipulated in the grid that emerged both in the diagnostic test and in the semi-structured interview with the teachers in multigrade schools. For this identification, the characteristics of the facets were first described, and then each teacher was analyzed using a traffic light to determine whether he or she complied fully (green), 50% (yellow) or did not comply with the indicator (red), in order to then obtain a number of teachers and establish them mathematically by means of a frequency, with the aim of analyzing the data both quantitatively and qualitatively. Also, these results from the qualitative component gave us some cognitive conceptions that allowed us to stipulate epistemic configurations associated with DMS and the personal meanings that each teacher had internalized, understanding that these epistemic configurations can be seen as an a-priori cognitive configuration.

In accordance with the above, the characteristics of the facets, their coding and the frequency obtained are shown in **Table 2**.

In relation to the ecological facet, it was perceived in this local study that the majority of teachers did not implement in their class designs a cross-cutting approach to knowledge with other areas of knowledge associated with DMS, even though they had pedagogical, didactic and institutional instruments (booklets, curricula, area plans ...) for the implementation and development of this cross-cutting approach in the classroom, to a certain degree preventing the mathematical object from being treated from a phenomenological perspective.

However, it can be observed in the frequency of the first indicator that 30% of the local study population

Table 3. Interactional facet

Component	Code	Characteristics	Frequency(%)
Interaction	IF1	He uses a variety of pictorial, argumentative and dynamic resources to involve and capture the attention of his students.	30
Teacher-student	IF2	Makes an appropriate presentation (clear, well organized, & emphasizing key concepts of topic) to his/her students.	20
	IF3	Recognizes and solves problem situations related to the mathematical object with their students (appropriate questions and answers are asked).	20
Autonomy	IF4	It expresses communication with the parent in an autonomous way, without any institutional obligation.	100
	IF5	Assumes with responsibility the importance of updating every day in order to propose solutions, examples and a variety of tools to solve mathematical situations related to the mathematical object of study in their classrooms.	70

demonstrates compliance with the curriculum adaptation component. This is reflected in their solid understanding of the mathematical object (content) and in the effective application of this knowledge in the classroom, aligning with the curricular guidelines of the educational institution.

Continuing with another element of this facet, related to EcF2, the result is that practicing teachers do not adequately integrate technological media in multi-grade schools in the teaching and learning of students, specifically of the mathematical content under study, which generates an imbalance in the opening towards innovative didactics. Despite this, 40% of teachers have an approach to this indicator, trying to make use of technological resources such as: tablets, televisions and platforms, with the aim of guiding and socializing tasks related to the notion of area, perimeter and decimal position. Finally, it should be noted that 30% of the teachers who teach in the rural context promote intra- and interdisciplinary connections through their work in the classroom, which subsequently benefit the understanding of mathematical content.

In the interactional facet, it stands out that 30% of the researched population demonstrates compliance with the interaction component, since the group of teachers uses a variety of pictorial, argumentative and dynamic resources in order to involve and capture the students' attention, fostering a participative and committed environment (**Table 3**).

On the other hand, 20% of the teachers stand out for implementing a bidirectional teaching-learning process in which they manage to make clear, organized and focused presentations on the relevant mathematical contents that make up the genesis of DMS, promoting a fluid dialogue with the students, from the posing of questions from real situations with respect to the mathematical object and pertinent, concise answers to the questions.

It is relevant to point out that all of the teachers in the study comply with the autonomy component. This aspect is evidenced by their ability to establish effective communication with parents in an autonomous manner, without being subject to institutional obligations.

Likewise, 70% of the population shows an outstanding commitment to this component by taking responsibility for constant updating. These educators recognize the importance of keeping up to date in order to provide solutions, examples and a variety of tools in the classroom, thus effectively addressing mathematical situations related to the object of study. This is reflected in their ability to present innovative solutions in their classrooms, such as creating models of meters, developing contextualized mathematical exercises, using worksheet platform to introduce modified guides, as well as implementing resources such as the abacus, the tangram, among others.

In the area of the mediational facet, it is encouraging to note that 100% of population studied demonstrates full compliance in the component of the use of material resources, whether manipulative (Legos, spherical objects, test tubes, simulation of metric objects, among others) or technological, in the classroom environment (**Table 4**).

As part of this percentage initially mentioned, it was identified that 30% of the teachers use the material resources with a didactic and pedagogical purpose, mainly routed to the understanding and construction of the concept of DMS, linking situations of the context close to the student, such as: measuring the classroom, measuring the court of the Educational Institution, measuring the amount of water irrigated to a coffee plant, the height of the plant, among others. This in turn allows the incorporation of diverse semiotic registers, for example: modelling, which provide the educational triad (teacher, knowledge, student) with a mechanism as a strategy for meaningful learning and internalization of mathematical content.

In relation to the efficient use of time in collective teaching and learning activities, it was observed that 60% of the teachers directed their classes under three factors: the first was associated with the learning levels of each of the students, largely due to the diversity of grades that the teacher has to handle in class; the second was the individualized advice given to the student in extra-class hours, providing a closer approach to the solution of concerns or questions that the student had;

Table 4. Mediational facet

Component	Code	Characteristics	Frequency(%)
Material resources (manipulative or technological)	MF1	Uses manipulative/technological materials that allow introduction of good situations, languages, procedures for teaching, & learning intended content.	100
	MF2	Applies definitions of mathematical object & articulates them with situations of context close to student (use of vegetable garden, use of field, classroom, among others), making use of concrete models, & visualizations.	30
	MF3	Demonstrates knowledge of the academic pathway to student learning outcomes, from a position as a purposeful and effective leader for the teaching and learning of mathematics.	30
Time (collective teaching & learning)	MF4	Allocate time for collective and individual teaching.	60

Table 5. Affective facet

Component	Code	Characteristics	Frequency(%)
Interests & needs	AF1	Shows willingness to maintain assertive communication with parents.	100
	AF2	Demonstrates interest in the cultural, social and economic context in which their students find themselves.	100
	AF3	It makes known through arguments and examples in class, the need to propose situations that allow us to value the usefulness of mathematics in everyday life and in the context close to the student.	80
Attitudes	AF4	Promotes participation in the activities proposed in class, either in the student or in the parent.	90
	AF5	Shows a willing attitude when answering the questions posed.	100
Emotions	AF6	Shows concern, demotivation when carrying out tasks related to the mathematical object.	70
	AF7	Shows willingness, cheerfulness when performing tasks related to the mathematical object.	100
	AF8	It highlights the aesthetic and precision qualities of mathematics.	20

and finally, the accompaniment of monitors, who were made up of students who had a mastery of the mathematical concepts dealt with in class, in order to transfer them to their peers.

With regard to the affective facet, it is evident that 100% of the population exhibits a marked willingness to maintain assertive communication with parents, showing a genuine interest in understanding the cultural, social and economic context in which their students find themselves (**Table 5**). Furthermore, 80% of the teachers emphasize the need to propose situations that allow them to value the usefulness of mathematics in everyday life and in the student's immediate environment, transmitting this importance through arguments and examples during the classes.

In relation to the attitudinal component, 90% of the teachers in this local study excel in encouraging active participation in the activities proposed in class, both by students and parents. This is largely generated by the teachers' interest in enabling parents to receive teaching in accordance with the topics being developed in the classroom, especially illiterate parents, and to support students in the construction of the mathematical concepts they wish to institutionalize. In addition, the entire population studied demonstrates a proactive attitude and willingness to address and resolve the questions posed, thus contributing to a participatory and enriching environment.

Regarding the emotional component, it is observed that 70% of the teachers show concern and demotivation when faced with tasks related to the mathematical object, this is due to two causes, one is that most of the teachers who guide mathematics in the rural context, are not generally graduates in mathematics but graduates in other areas of knowledge (social, Spanish, early childhood education and physical education), engineers, public accountants, among others, and another cause is because the mathematics teacher who is more familiar with the mathematical object does not have an internalization of this.

However, 100% of the population studied showed a positive disposition and showed joy in wanting to learn methodological strategies that allow them to tackle tasks related to DMS, highlighting the importance of generating an environment that is conducive to mathematical learning. This leads to 20% of the teachers highlighting the aesthetic and precision qualities of mathematics, recognizing its impact on the development of study and highlighting aspects that go beyond simple problem-solving.

With regard to the cognitive facet, it is relevant to highlight that 30% of the teachers in the population studied demonstrate outstanding compliance in the component of prior knowledge (**Table 6**). This group is distinguished by attributing personal meanings to the mathematical object DMS, based on mental representations associated with numerical, algebraic,

Table 6. Cognitive facet

Component	Code	Characteristics	Frequency(%)
Interests & needs	CF1	Attributes personal meanings to the mathematical object DMS, based on their mental representations.	30
	CF2	Demonstrates prior knowledge necessary for the solution of the problem situations posed in the study.	20
Attitudes	CF3	Has a conceptual, propositional, & procedural understanding of mathematical object DMS.	20
	CF4	Shows an appropriation of argumentative, problem-solving, communicative, & modelling competence (when establishing a model as a solution to problem situation) when solving situations related to mathematical content.	20

Table 7. Epistemic facet

Component	Code	Characteristics	Frequency(%)
Situations-problem	EF1	It presents an articulation between the contextualized problem situations posed and their mathematization.	30
	EF2	It proposes situations for the generation of real problems.	40
Languages	EF3	Uses different registers of representation (verbal, graphic, symbolic, among others) making conversions between them.	30
	EF4	Uses age-appropriate language to communicate with learners.	30
Rules (definitions, propositions, & procedures)	EF5	Applies clear and correct definitions and procedures adapted to the respective educational level in problem situations.	20
	EF6	It is clear between the various systems of measurement units (capacity, volume, length and area).	0
	EF7	It proposes situations in which the student has to generate definitions or procedures in a constructive way.	30
Arguments	EF8	It states appropriate explanations, checks, & arguments to justify its procedures.	20
Relations	EF9	Articulates personal meanings of mathematical objects with institutional meanings.	20

graphical and symbolic registers. With regard to the learning component, it was identified that 20% of the population in this local study stand out as demonstrating a deep and multifaceted understanding of the mathematical object DMS.

These teachers show conceptual, propositional and procedural understanding, indicating a solid cognitive base in relation to the mathematical content.

In addition, it is observed that 20% of the teachers have a significant appropriation of key competences, such as argumentative, problem-solving, communication and modelling (by establishing models as a solution to problem situations) when dealing with contexts related to the mathematical content. This group stands out for their ability to effectively apply these competences in their pedagogical practice, thus enriching the teaching and learning process.

In the area of the epistemic facet, 30% of the teachers demonstrate a remarkable ability to articulate the proposed contextualized problem situations with their mathematics in which versatile skills such as the use of semiotic registers and the articulation between the mathematical content and its practical application are employed (**Table 7**). Likewise, in the application of the diagnostic test and the semi-structured interview, 30% of the teachers use appropriate language to communicate with the students, adapting it to their age and level of understanding, stimulating and generating definitions and procedures in a constructivist manner in the students, promoting a participatory and active approach to learning mathematics.

In terms of performance in solving problem situations, 20% of the teachers excel in applying clear and correct definitions and procedures, adapted to the respective educational level. However, the need to deal clearly with the various systems of units of measurement (capacity, volume, length and surface area) is noticeable, as none of the participants demonstrated an optimal understanding in this respect. This statement stands out when teachers provide explanations, verifications and arguments to justify their results. Furthermore, they manage to effectively articulate the personal meanings they have of mathematical objects with institutional meanings, thus demonstrating an approach to the understanding and internalization of DMS.

Thus, within this facet, three epistemic configurations were evidenced that allowed us to identify the personal meanings of three of the ten teachers who participated in this research. These meanings considered formal aspects (specific to mathematics) and empirical aspects (related to the teacher's ontological and epistemic praxeology). It is important to note that the responses of the remaining

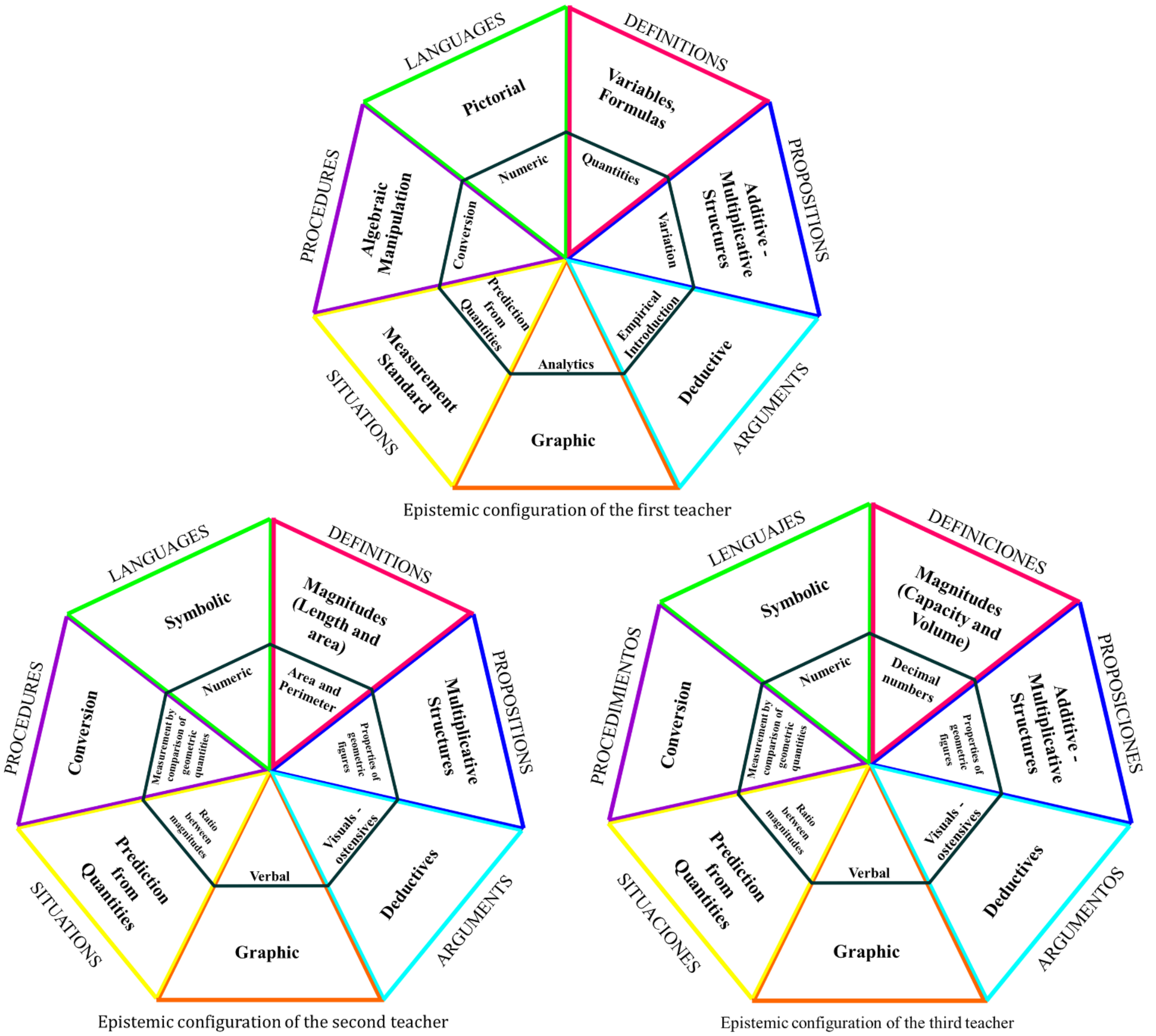

Figure 3. Epistemic configurations (personal meanings) of practicing teachers in rural context (Source: Authors' own elaboration)

seven teachers did not allow for the realization of an epistemic configuration.

Figure 3 shows that the first teacher in the research defines DMS as a measurement standard in which additive and multiplicative schemes are used as a tool for carrying out conversions between units of measurement and languages associated with pictorial and numerical representations, which facilitate the understanding of the problem situations posed in the diagnostic test, resulting in a model provided by the teacher in an algebraic form.

Continuing with the second teacher, his conception or idea of the mathematical object is directly associated with the notions of the magnitudes length and surface area, relating mathematical processes such as algorithmic procedures and comparison of quantities by means of whole numbers, under the concepts of area and perimeter in which he used symbolic and numerical registers.

Finally, with the third teacher, the meaning of DMS is routed to the magnitudes of volume and capacity, making use of decimal numbers, multiplicative structures and units of measurement.

CONCLUSIONS & DISCUSSION

The aim of this research was to answer the following question: What personal meanings do teachers in a rural context have in relation to DMS?

For this purpose, the data were analyzed using the six facets of the didactic dimension of DMK model. We focused on the epistemic and cognitive facets, since these are the ones that are directly related to the personal meanings of the rural context teachers.

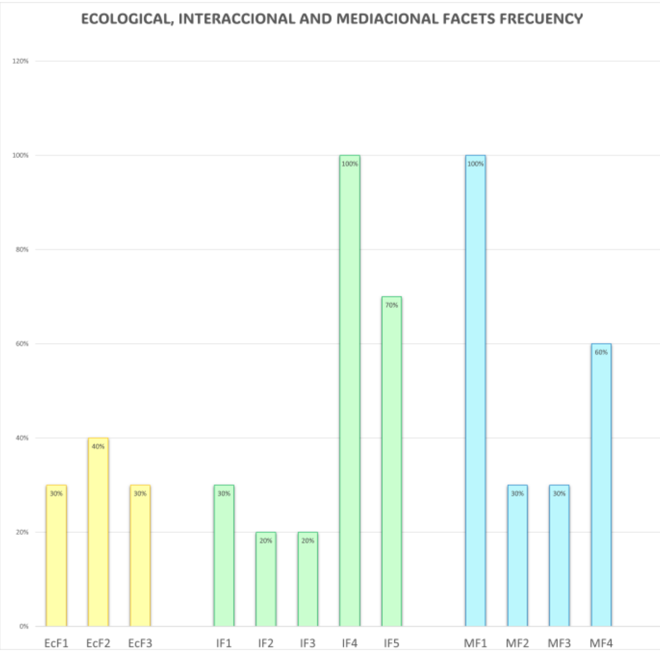

Figure 4. Ecological, interactional, & mediational facet frequency (Source: Authors' own elaboration)

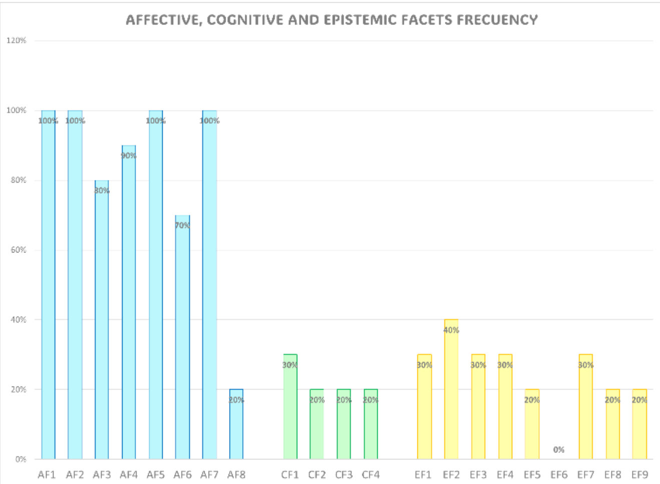

Figure 5. Affective, cognitive, & epistemic facets frequency (Source: Authors' own elaboration)

However, we have drawn a route to make a comparison between these facets, dividing them into two groups: the first group (**Figure 4**) is made up of the ecological, interactional and mediational facets and the second group (**Figure 5**) is made up of the affective, cognitive and epistemic facets.

It is necessary for the teacher to have a closer approach with the parents, associated with the teaching of the concepts oriented in class, so that there is a triangulation of knowledge between the institutional, the personal and the family, since in the particular case of students who belong to the rural context, one of their greatest supports is the family link, due to economic and contextual difficulties in accessing the internet or virtual sources of information. This was evidenced in the research by the fact that some of the teachers did not design or prepare their classes in advance, to be socialized with the immediate environment, even if it was written or stipulated in a curriculum or area plan. Also, this study reveals that teachers, when teaching in the rural classrooms evaluated, use both manipulative materials and technological resources (see **Figure 4**).

Likewise, they introduce situations, languages and procedures into the educational environment that are adapted to the context, facilitating the teaching-learning process of the desired content, being a proactive and effective leader to guide mathematics education process.

However, it is observed that most teachers do not apply the definitions of the mathematical object DMS, and as a consequence, do not establish a connection with the student's everyday situations, such as the use of the vegetable garden, the court or the classroom, among others. This challenge could be overcome by using concrete models that would offer a clearer visualization of this mathematical object. In addition, it is important to highlight that most of the teachers demonstrate an adequate distribution of time for both the collective and individual teaching process (see **Figure 4**).

However, the comparison of the second group is presented in **Figure 5**. According to **Figure 5**, educators in the rural context show a deep interest in understanding the cultural, social and economic context in which their students find themselves. They use arguments and examples in class to highlight the relevance of these contexts, emphasizing the usefulness of mathematics in everyday life and in the student's immediate environment. However, a concern is perceived in the research when analyzing that the teacher largely addresses the affective component of the student and neglects the cognitive component (see **Figure 5**). This study showed that one of the causes underlying this is the lack of mastery of the concepts associated with DMS, their personal meanings and their connections to previous knowledge associated with mathematical processes and mathematical competences.

Finally, in the epistemic facet, it was evident that most of the teachers lacked an adequate articulation between the contextualized problem situations posed and their mathematics. However, it is worth noting that some of the teachers, specifically one of them came up with a mathematical model in which they made known what their mental representation of DMS was like, making use of the representation registers such as verbal, graphic and symbolic, carrying out a conversion between these registers, by means of the relationship between two or more representations of a mathematical concept. This is consistent with what was reported in the research by Campo-Meneses et al. (2021), although with another mathematical object.

On the other hand, it is observed that 90% of the multigrade school teachers do not present clarity in all the systems of units of measurement (capacity, volume, length and surface area), if not in one or at most two of these magnitudes, which makes us reflect that it is important to generate definitions, as procedures in a constructivist way in which the teacher provides explanations, verifications, and arguments with intra and extra mathematical connections (Hernández-Yañez et al., 2023). Therefore, this research has an impact from an inclusive mathematics education based on the training of teachers in rural areas, which has an impact on the teaching and learning of mathematics. Thus, some of the contributions made by the study is to show what conceptual constructs the rural context teacher has and how the facets intervene when making mental records in the solution of tasks related to DMS. The research also provides an identification of the possible difficulties or obstacles of a semiotic, ontological and pragmatic nature through the characteristics of the facets, which the multigrade schoolteacher develops in the teaching of this mathematical object.

Recent research, such as those conducted by Amaya De Armas et al. (2016) and Pino-Fan et al. (2018), has shown a growing interest in understanding the necessary knowledge that mathematics teachers require to teach effectively. In this sense, the use of the six facets of the didactic dimension of DMK (Pino-Fan et al., 2015) has become relevant, both in initial and in-service teacher training, especially in areas such as probability and in the curriculum in general.

These studies have revealed that detailed analysis of the facets of DMK offers multiple benefits. Firstly, it has been observed that by considering the ecological facet, focusing on curricular materials, higher quality teaching can be ensured by selecting resources that are effective and appropriate for promoting student learning. In addition, teachers' professional development is strengthened by the contributions of the epistemic and cognitive facets, which provide them with tools to critically reflect on their practice and continuously improve their performance (Martínez, 2021; Pino-Fan, 2014). In the context of our research, the application of the diagnostic test and the semi-structured interview allowed us not only to assess the level of understanding of DMS, but also to identify areas for improvement in teaching practice. The characteristics of the six facets of DMK, in addition to serving as cognitive and epistemic instruments, showed their usefulness in mediational, interactional and affective aspects. In this way, teachers were able to reflect on their strengths and weaknesses in the teaching process, with the aim of improving their work as trainers and ensuring meaningful learning for their students, in accordance with the educational standards proposed for multi-grade schools.

With regard to the personal meanings of the multi-grade schoolteachers, the idea of epistemic configuration was used, since this construct allows us to visualize elements that the teacher handles or has incorporated in his or her cognitive part and transmits to the students in the process of teaching DMS. These elements allowed procedures, situations, arguments, definitions, languages and propositions that the teacher makes when solving a real context situation explicitly associated with the magnitudes of capacity, volume, length and surface area to emerge.

In the analysis of the personal meanings of teachers in the rural context, a ten-item questionnaire was used as an instrument, based on the validation of scientific articles (Ávila & García, 2020; Burgos et al., 2019; Sua Tarazona, 2023) in which task designs for the case of length and surface area were presented by experts in the field of rural mathematics education, who expressed the need for the teacher to present a mastery in the recognition of the notions and differences between the volume and capacity of an object.

Thus, from the above we can conclude that the personal meanings of the rural teachers in this local study, on the concept of DMS is closely linked to measurement patterns in which they procedurally use multiplicative structures and connect these conceptions with the register of graphic representation as numerical. Also, associating their achieved meaning with measurement by comparison of geometric magnitudes in which they declared the increase or decrease of a quantity. However, in relation to the global personal meanings generated by seven teachers in multigrade schools, they generalized the concept of DMS as the transformations carried out through unit conversion tables in each magnitude, limiting its internalization and thus subsequently its transition to their students.

Finally, future research could study the personal meanings associated with DMS that teachers have and put into practice in their classrooms. In addition, the results of this research could be contrasted with the personal meanings of teachers in the urban context.

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Ethical statement: The authors declare that the study was approved by the coordinator of the line in Mathematics Education of the Master in Educational Sciences of the University of Quindío (December 2023). Written informed consent was obtained from the participants.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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