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Prospective primary school teachers' mathematical knowledge of sampling

Alondra Stuardo-Aguayo [1], María Magdalena Gea [1*], Nuria Begué [2], Carmen Batanero [3]

[1] Department of Mathematics Education, University of Granada, Granada, SPAIN
[2] Department of Mathematics (Field of Mathematics Education), University of Zaragoza, Zaragoza, SPAIN
[3] Research Group FQM126, University of Granada, Granada, SPAIN

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Abstract

Sampling is a fundamental stochastic concept that bridges statistics and probability. Due to its importance, some elementary sampling ideas have been included in the latest Spanish primary education decrees; however, teachers may not be familiar with the topic, and research on the sampling knowledge of teachers is scarce. To fill this gap, the aim of this study was to assess common and advanced mathematical knowledge of sampling of 105 prospective primary school teachers by analyzing their responses to a questionnaire with open-ended questions. Common content knowledge was assessed by asking for definitions and examples of sample and population and by discriminating between correct and incorrect sampling methods. Advanced knowledge was tested by asking for an estimation of the population mean from the sample mean and of the population and proportion in a new sample from the sample proportion. Results indicated better common content knowledge than previous studies and gaps in advanced knowledge. Most participants had a multi-structural understanding of sampling concepts, where several aspects were described but not related to each other. We conclude that there is a need for better preparation of prospective primary school teachers' sampling knowledge considering the current curricular deepening on this topic, and new research should be oriented toward designing and evaluating related teaching actions.

Keywords: sampling, sampling methods, prospective primary school teachers, assessing understanding

INTRODUCTION

Sampling is the basis of statistical inference, which allows building knowledge about a population using only data obtained from the population as a representative sample. This is part of statistical literacy because of the many sampling applications in everyday life (Batanero et al., 2019; Muñiz-Rodríguez et al., 2020; Sharma, 2017). For example, a doctor examines a blood analysis from a sample of a few drops of blood, or media news reports the voter intention by an opinion poll with a particular sampling frame.

In addition, sampling implies a connection between statistics and probability by what Burrill and Biehler (2011) and Heitele (1975) considered as a fundamental stochastic idea that can be taught at different levels of formalization across all school levels. Moreover, sampling processes appear both in the frequentist approach to probability and in the didactic use of simulation, which is now recommended for informally introducing ideas of inference (e.g., van Dijke-Droogers et al., 2020).

Echoing this prominence, recent curricular documents have included the topic of inference as part of stochastic sense-making in different cycles of primary education. In Spain, the first cycle (6-8 years) introduces the application of appropriate strategies for collecting data, and in the second cycle (8-10 years), students are expected to formulate conjectures from the data collected and analyzed (Ministerio de Educación y Formación Profesional [Ministry of Education and Professional Training] [MEFP], 2022). Concerning intuitive ideas about sampling methods, in the third cycle (10-12 years), the different stages that constitute the research process, including data collection, are

✉ alondrastuardo@correo.ugr.es ✉ mmgea@ugr.es **(*Correspondence)** ✉ nbegue@unizar.es ✉ batanero@ugr.es

Contribution to the literature

- Research on the sampling knowledge of prospective teachers is scarce, even though curricular documents in many countries include the topic of inference as part of stochastic sense-making in primary education.
- The comparison of prospective primary school teachers' responses and arguments to validated items from various studies adds significant new findings to the literature as well as analyzing the levels of their sampling understanding.
- Practical implications of the study include the need to reinforce teachers' sampling knowledge. Thus, we propose directions to enhance teachers' education programs.

incorporated. In addition, in this cycle, the frequentist approach of probability is introduced, which is related to sampling, and completes the classical meaning that was incorporated in the second cycle for simple random phenomena.

These new guidelines require the adequate preparation of prospective primary school teachers with mathematical and didactic knowledge of the ideas underlying the sampling work. This includes intuitive ideas of population and sample, random methods, sampling bias, the relationship between the sample and population mean and proportion, and their variability (Batanero et al., 2019), which we evaluate in the prospective participants' teachers in this research. Although these ideas may appear simple, previous research analyzing subjects' understanding of the ideas underlying sampling has revealed difficulties in understanding and reasoning biases. For example, Begué et al. (2017) and Castro-Sotos et al. (2007) suggested that students of different ages put too much confidence in small samples and do not understand the role of sample size in controlling sample variability. Harradine et al. (2011) found that students confused the distribution of a variable in a population with the distribution of data in a sample thereof. These studies were considered to support our research and the selection of tasks in the questionnaire. Although research on mathematics teacher education is extensive (Hwang & Cho, 2021; Llinares, 2023; Miyakawa, 2022), little is known about the sampling knowledge of teachers. To provide information in this regard, this paper aimed to assess the mathematical knowledge of sampling in 105 prospective primary school teachers. This study is significant because it provides information regarding points on which teachers' education should be reinforced. In the next sections, the background and theoretical framework, methods, and results of the study are presented, followed by the conclusions.

PREVIOUS RESEARCH

Most research on sampling has been conducted with students and a few with prospective teachers. In general, research conducted with students analyzed students' understanding of the sampling concept, its properties, and the different sampling methods. Research on prospective teachers mainly studied their comprehension of sampling variability, the relationship between the sample and population proportion, and the role of the sample in obtaining informal inferences for a population. Below, we describe the outcomes of these studies, and some tasks considered in our research to construct the questionnaire and interpret our results.

Research With Students

Understanding of the sampling concept was analyzed by Watson and Moritz (2000) with 62 students in the 3rd and 6th grades of primary school and 3rd year of secondary school (8, 11, and 14 years old). The authors asked the students what they understood from the word *sample* and requested examples of situations in which it is necessary to use samples. From their answers, the authors differentiated various levels of development in understanding sampling based on part of the structure of observed learning outcomes (SOLO) taxonomy (Biggs & Collis, 1982), which is described in the theoretical framework. Younger students (8 and 9 years old) exhibited primitive sampling ideas derived from their everyday experiences and were confident in drawing conclusions using small samples. In contrast, older students (14 and 15 years old) appreciated the variability of the population and felt the need for larger and more representative samples, but sometimes failed to detect sampling bias.

Jacobs (1999), Meletiou-Mavrotheris and Paparistodemu (2015), Watson (2004), Watson and Kelly (2005), and Ruiz-Reyes (2020) analyzed the understanding of sampling methods. Jacobs (1999) reported that although many 4th and 5th primary school students (10 and 11 years old) accepted the advantage of random sampling, the tendency was to prefer stratified sampling to random sampling. Detecting bias in convenience sampling methods was also difficult, possibly because of concerns about ensuring fair and representative samples.

Watson (2004) interviewed 38 students of 8-14 years of age on the different ways of selecting samples. Their results showed a preference for biased samples over random or stratified random sampling. A similar result was obtained by Watson and Kelly (2005), with 639 students in the 3rd and 5th grades of primary school and in the 1st and 3rd grades of secondary school (13 and 15 years old), as very few identified sampling biases or considered random sampling to be appropriate. Meletiou-Mavrotheris and Paparistodemu (2015) assessed 69 primary school students in grades 4-6 (9-12 years) with questions similar to those in Watson's (2004) work. The authors indicated that their students did not consider simple random sampling appropriate because they felt that it could result in a sample that did not represent the diversity of the population. Therefore, they indicated a preference for stratified or convenience sampling. Similar results were obtained by Ruiz-Reyes (2020), with a sample of 1200 secondary and high school students, where only 10.1% and 43.8% of students correctly identified biased methods, while 56.8% correctly recognized the stratified method as adequate and 40.8% correctly considered the random method as correct.

Research With Prospective Teachers

Abu-Ghalyoun (2021) and Begué et al. (2023) investigated prospective secondary school teachers' understanding of sampling variability and reported that not all teachers understood the effect of sample size on sampling variability; moreover, some believed that big samples were more variable than small samples.

Regarding the relationship between the sample and population proportion, Gómez-Torres et al. (2018) requested the total number of elements in a finite population from the sampling proportion to 157 prospective primary school teachers and obtained 20.4% correct answers. She also requested the expected proportion of cases with a characteristic in the population and obtained 20.4% correct answers.

The role of sampling in obtaining informal inference for a population was examined in Batanero et al. (2022) and de Vetten et al. (2019). Batanero et al. (2022) analyzed how a sample of prospective secondary school teachers estimated the composition of a ballot box based on data from 1,000 ballot box draws. Although most teachers built a model of ballot box composition consistent with a given sample, few used it to predict the outcome of the next draw. de Vetten et al. (2019) analyzed the responses of 722 prospective primary school teachers in the Netherlands to five multiple-choice questions regarding sampling in the context of informal statistical inference. When asked to make a generalization beyond the data, most participants only described the data and did not appear to understand how a representative sample can be used to make inferences about a population.

To complement these studies, in this research, we analyze the mathematical knowledge of the following content on sampling in a group of prospective primary school teachers: sample and population; sampling methods; sample mean, its distribution, and use in estimation; sample proportion, its distribution, and use in estimation. In the following sections, we describe the background of the study, methodology, results, and conclusions.

THEORETICAL FRAMEWORK

We based our research on two theoretical frameworks. First, the didactic-mathematical knowledge of the teacher (DMKT) model allowed identify the different components of teacher knowledge, two of which (common content knowledge and advanced content knowledge) were aimed in this study. Second, the SOLO taxonomy enabled us to provide the different sampling reasoning levels.

Teacher Knowledge

Nowadays, there is a growing body of research on mathematics teacher knowledge and education based on different theoretical models (e.g., Castro & Toro, 2023; Ledezma et al., 2023; Llinares, 2023). The DMKT model employed in this study was described by Godino (2009, 2024); Godino et al. (2017), and Pino-Fan et al. (2015).

The DMKT model includes common knowledge (mathematical content that the teacher must teach about the subject) and advanced knowledge (broader mathematical content knowledge, which allows articulating its teaching at higher educational levels). In addition, the teacher needs didactical-mathematical knowledge, which in the DMKT model is described by the following facets: epistemic (knowledge of the meaning of mathematical objects and practices for their teaching), ecological (relationship of mathematical content with other subjects in the curriculum and with society), cognitive (knowledge of students' learning, difficulties, and reasoning), affective (knowledge and management of students' attitudes, beliefs, and emotions), mediational (knowledge of didactic resources, including technological ones), and interactional (management of classroom discourse).

To fulfill our aims, our study focused on understanding various elementary sampling contents, covering common and advanced knowledge, which are described in detail in Section 4. In addition, to further understand the participants' sampling knowledge, we also analyzed their reasoning level regarding sample size and sampling variability, as is described in the next section.

Levels of Understanding Sampling

Two items included in the questionnaire enabled us to determine levels of understanding of sampling using the SOLO taxonomy. This taxonomy was proposed by Biggs and Collis (1982) and defines levels of increasing complexity for assessing the quality of responses to school items. The model comprises the following levels:

- **Pre-structural level:** Not understanding the content. Students do not respond or provide

Table 1. Understanding levels of the concepts of sample, population, and sampling variability

Level	Concept	
	Sample and population (Watson & Moritz, 2000)	Sampling variability (Ko, 2016)
Pre-structural	Confusing the sample and population or giving an incorrect definition.	Students confuse the data in a sample with sample statistics. Although they believe that different samples can have different statistics, they do not recognize patterns in the sample statistics.
Uni-structural	Focus on only one aspect of the sample, e.g. being a subset of the population or part of a study.	Students notice the spread of sample statistics (variability) but do not recognize their central tendency (representativeness).
Multi-structural	Several aspects associated with the sample, e.g., being a subset of the population and being part of a statistical survey.	Sample representativeness and variability are allowed but are not related to sample size.
Relational	All elements of the sample required for a study are related, including the purpose of the sample and its relationship to the population.	Representativeness and sampling variability are allowed and are related to sample size.

incorrect answers, focusing on irrelevant aspects of the item.

- **Uni-structural level:** Students focus on a single relevant aspect of the content.
- **Multi-structural level:** Students focus on two or more correct aspects of the content that are analyzed separately but are not interrelated.
- **Relational level:** The student identifies and interrelates several correct aspects, thereby integrating the information into a comprehensive whole.
- **Extended level of abstraction:** Responses that demonstrate the structure of content knowledge using more abstract concepts and processes than those required for the item.

Watson and Moritz (2000) used this taxonomy to assess the understanding of sample and Ko (2016) to assess understanding of sampling variability. Table 1 shows the expected characteristics of the responses related to these two concepts for levels used in our study (the level reached in the conception of sample and population is assessed in item 1 and the one corresponding to sampling variability in item 3). We used the classification of Watson and Moritz (2000) and Ko (2016) to analyze the level of sampling understanding among the participants.

METHOD

The sample consisted of 105 prospective primary school teachers in their second year of primary education degree at the University of Granada, Spain. The instrument was implemented as an instructional activity in mathematics teaching and learning in two groups of prospective teachers by a researcher who acted as the teacher.

Although admission to a bachelor's degree requires taking a university entrance exam and having completed high school, it is possible to enter through a high school specialization without mathematics content. However, all the students had studied statistics and probability during their compulsory secondary education (CSE), including elementary sampling concepts. Considering ethical considerations, we obtained informed consent from the participants and ensured the confidentiality of their data.

Questionnaire

The questionnaire (reproduced in **Appendix A**) was constructed by the authors using four items taken from various studies in which they had already been validated. All items follow the open-ended format question because it is preferable when the item has more than one correct solution and is focused on higher-order thinking skills (Altıntaş, 2022). Below, we describe the content of each item.

Item 1 was adapted from Watson and Moritz (2000), who administered it to 62 students in the 3rd and 6th grades of primary school (8 and 12 years old) and 3rd year of secondary school (15 years old). It was also used by Meletiou-Mavrotheris and Paparistodemou (2015) with 69 students in the 4th to 6th grades of primary school (9-12 years old).

Item 2 was used by Jacobs (1999) with a small sample of 4th and 5th year primary school students (9-10 years old), Meletiou-Mavrotheris and Paparistodemou (2015), Watson and Kelly (2005) with 638 students from 3rd and 5th year primary school and 1st and 3rd year secondary school (13 and 15 years old) and Ruiz-Reyes, (2020) with 1,200 students from 4th year secondary school (16 years old) and high school (17-18 years old). Students were asked to decide whether various sampling methods were appropriate.

Item 3 is adapted from Ko's (2016) research, who administered it to 130 students from 5th grade of primary school to 2nd year of secondary school (11-14 years old) and analyzes the understanding of the sample mean and its distribution, its representativeness, and sampling variability, as well as the estimation of the population mean from the sample mean.

Table 2. Mathematical content assessed in the questionnaire

Mathematical knowledge	Sampling content		I1	I2	I3	I4
Common content knowledge	Sample and population	Intuitive definition	x			
		Example	x			
	Sampling methods	Random sampling		x		
		Stratified sampling		x		
		Restricted population		x		
		Bias in sampling		x		
Advanced content knowledge	Sample mean, its distribution, and its use in estimation	Representativeness			x	
		Estimation of the population mean			x	
		Variability			x	
		Relationship with sample size			x	
	Sample proportion, its distribution, and its use in estimation	Representativeness				x
		Estimation of the population proportion				x
		Estimation of population size				x
		Variability				x

Gómez-Torres et al. (2018) used item 4 with prospective primary school teachers and Ruiz-Reyes (2020) with secondary and high school students. It analyzes the understanding of the sample proportion, its distribution, representativeness, and variability, its use in estimating the proportion, and the size of a finite population from the sample proportion.

All items assessed knowledge of the different sampling components, as shown in **Table 2**. The first two items correspond to common content knowledge because they assess topics that can be addressed in primary education and were taken from research on primary school students. The next two items correspond to advanced content knowledge, which is more appropriate for secondary education because they are derived from research with secondary school students.

The reliability analysis of the questionnaire, taking as separate variables each of the responses and arguments to the different sections (in total 14 variables), resulted in a Cronbach's alpha = .645. Although this value is not too high, Loewenthal (1996) indicated that a value equal to or higher than .6 is reasonable when the number of items is small. This is the case in our questionnaire, in which items assess different contents because Cronbach's alpha takes higher values only for very homogeneous items.

Generalizability coefficients, which employ repeated-measures factorial analysis of variance to estimate the relative contributions of different sources of variability to the overall measurement error (Brennan, 2001), were also computed. In generalizability theory, the total variance of scores obtained from the responses to a questionnaire is divided into different sources of variability to consider different sources of error in the scores, such as individual scores, students, items, or conditions applied. In this way, the total variance can be decomposed into different parts to compute the coefficients that generalize the idea of reliability (Medvedev & Siegert, 2022).

In our case, we have considered, on the one hand, the variability of responses due to the items; because each item evaluates a different type of knowledge, this variability is high; therefore, the possibility of generalizing to other similar items is moderate because Gi = .65, which is close to the value of the Cronbach's alpha. On the other hand, we estimated the generalizability of students' scores and obtained a very high generalizability Gs = .97. This means that the results will be stable when using the same questionnaire with other similar students to those in the sample and moderate when proposing different sampling tasks to the participants in the study.

Once the written responses to the questionnaire were collected, we performed a content analysis (Krippendorf, 2018), which is a method for categorizing, classifying, and identifying patterns in qualitative data. This method comprises the following steps:

1. We first selected the content to be analyzed, i.e., the set of individual written responses of the 105 participants to each questionnaire item. We defined the units and categories of analysis.
2. Once the responses were read and we became familiar with them, we classified the responses and created categories for coding.
3. The initial coding was reviewed by different authors, and the discordant cases were discussed until a consensus was reached.

The results are presented below, differentiating between common and advanced sampling knowledge. We included examples of correct responses, where participants have been coded as Pxx, where xx is the order of the participant in the group.

RESULTS IN COMMON CONTENT KNOWLEDGE

As indicated, the first two items assessed common content knowledge because they considered intuitive knowledge that primary school students should share.

Table 3. Frequency and percentage of students according to the correctness of their definitions and examples (item 1)

	Definition				Example	
	Sample		Population			
	Frequency	Percentage (%)	Frequency	Percentage (%)	Frequency	Percentage (%)
Correct	72	68.6	64	61.0	52	49.5
Incorrect or imprecise	28	26.7	33	31.4	46	43.8
No response	5	4.8	8	7.6	7	6.7

Table 4. Frequency (F) and percentage (P) of arguments for sampling methods considering their correctness (item 2)

Sampling method	Is it correct?	Type of argument								Total	
		Correct		Partly correct		Incorrect		No argument			
		F	P (%)[1]	F	P (%)[1]	F	P (%)[1]	F	P (%)[1]	F	P (%)[2]
Stratified (María)	Yes	60	76.9	17	21.8			1	1.3	78	74.3
	No					27	100			27	25.7
Biased (Luis)	Yes					3	100			3	2.9
	No	78	77.2	21	20.8	2	2.0			101	96.2
	NR*							1	100	1	1.0
Random (Rocío)	Yes	35	77.8	9	20.0			1	2.2	45	42.9
	No	10	17.2			48	82.8			58	55.2
	NR*							2	100	2	1.9
Intentional (Ana)	Yes					11	100			11	10.5
	No	89	95.7	4	4.3					93	88.6
	NR*							1	100	1	1.0

Note. *NR: No response; [1]Percentage in relation to the row; & [2]Percentage in relation to the total participants

Understanding Sample and Population

Table 3 presents the responses to item 1 (definitions and examples of the concepts of sample and population). Imprecise definitions contain only one aspect of the concept, e.g., "Yes, sample is a quantity, and population is the location where we took the sample" (P11). An example of an incorrect definition is "Sample, the set of possible cases, i.e., the sample space" (P42).

The main finding was that most participants correctly defined both concepts, although 43.8% provided incorrect examples. There was a significant proportion of imprecise definitions in both the sample and the population concept and examples, while only a few participants did not respond.

When we compared with Ruiz-Reyes (2020), she received 76.1% correct answers on the sample definition, a percentage slightly higher than ours, even when she analyzed secondary school students. However, neither did she ask about the concept of population nor provide any examples. We detail the observed levels of understanding in SOLO taxonomy for these two concepts later.

Acceptable Sampling Methods

In **Table 4**, we present the responses and arguments in item 2 on whether the proposed sampling methods are appropriate. In this item, 94.3% and 88.6% of the participants rejected Luis' and Ana's biased sampling methods, respectively, with correct or partly correct arguments. Maria's stratified random sampling was accepted by a large majority, although slightly more than a quarter of the participants did not consider it appropriate. Rocío's simple random sampling method, which is correct only when the school is restricted to secondary school students, was correctly or correctly partly considered by 41.9% of participants and incorrect by more than half of the sample. A significant outcome was that most participants distinguished the appropriate sampling methods, except for the example in which the population was restricted (Rocio).

The results obtained were better than those of Ruiz-Reyes (2020), in which 10.1% and 43.8% of secondary and high school students considered Luis' and Ana's biased methods, respectively, to be correct. They were also better than those of Watson and Kelly (2005), in which only 5.8% of students in their last year of primary and compulsory secondary school indicated that simple random sampling was appropriate. Consequently, the prospective teachers' knowledge of appropriate sampling methods was better than those found in research with students.

Arguments used to justify the sampling methods

A further analysis of the arguments used to justify responses to item 2 is described below.

Correct justification: Maria's stratified random sampling was considered correct because it was random and took the same number of students from each grade (see P34). Rocío's method (simply random) was considered correct if the school only included secondary students (P13) or incorrect if the school also had high school students (P8). Luis and Ana's biased sampling methods were considered incorrect because the sample was not representative (P5). Therefore, these arguments demonstrate an understanding of the characteristics of appropriate sampling methods.

> P34: Yes, because she randomly chooses students from each grade. She not only focuses on one grade but also on students with certain characteristics. In addition, she chooses the same number for each grade (justification for Maria's sampling).

> P13: Yes, it is correct if the school only includes secondary school students because if there were more grades, the survey would be affected by people outside the selected range (justification to Rocío's method).

> P8: No, since in a secondary school, in addition to the four years of compulsory school, there are two high school years, and we are only interested in compulsory secondary students (justification for Rocío's method).

> P5: No, […] because the members of the reading club do not represent the entire student population, as they are more interested in reading than the remaining students (justification for Luis' method).

Partially correct justification: These participants include errors or biases in their responses that are mixed with correct sampling ideas. For stratified random sampling (Maria), some students preferred (P9) contrast representativeness to random selection. In simple random sampling (Rocío), it is either indicated that stratified sampling is necessary, or it is not specified that the sample should be restricted to compulsory secondary school grades (P28). In Luis and Ana's biased sampling, some students identified the bias, but they considered the method incorrect for not having stratified the sample by grade (P57).

> P9: Yes, because you have drawn a representative sample from each year. Even this was performed randomly (justification for Maria's sampling).

> P28: It is correct because it is a representative sample as it is large, although it should be selected by course to have a smaller error, so it is not entirely correct (justification for Rocío's sample).

> P57: I think it is not correct because not all the students who read books are in the reading club, and it is not known if those who are part of the club are distributed by year (justification for Luis' method).

Incorrect justification: Regarding stratified random sampling (Maria), the arguments that do not consider random (P23) or stratified (P38) selection to be acceptable are incorrect. In simple random sampling (Rocío), some students, in addition to not indicating that it is necessary not to include high school students, demonstrated their distrust of the method (P14). Finally, the arguments that do not detect bias in the sampling methods of Luis or Ana are incorrect (P29).

> P23: You have used a random procedure. It is not correct because you do not obtain accurate data (justification to Maria's method).

> P38: No because it is not a random sample because 15 students were chosen from each class (justification for Maria's method).

> P59: No, because it is a random sample, it can happen that no students from a grade are included, so the results of the survey would be more inaccurate (justification for Rocío's method).

> P29: Yes, because it requires people who read really to obtain more accurate data (justification for Luis' method).

In **Table 4**, we observe that most of the arguments supporting Maria's stratified random sampling were correct, although some correct answers were linked to partially correct arguments by opposing representativeness and randomness and to incorrect ones by considering Maria's sample was too small. The rejection of Maria's method was linked to incorrect arguments, generally because the randomness of the sample elements was not accepted. We highlight the fact that this tendency was not observed in Ruiz-Reyes (2020), who reported that the rate of correct arguments was lower (56.8% of her students gave correct arguments and 4.8% partially correct).

The correct arguments for simple random sampling (Rocío) were divided into those that accepted the method and those that rejected it. The second group rejected Rocío's sample because they assumed that high school students could also be included. Just one-third of the participants gave the correct answer and argument, and the proportion of participants with incorrect answers was the highest among the items. In Ruiz-Reyes (2020), 40.8% of the arguments supporting random sampling were correct, and 13.3% were partially correct. However, in her item, there was no need to restrict the population, which may explain the difference from our results.

Regarding biased sampling methods (Luis and Ana), most participants rejected these methods. Furthermore, most of the arguments given to support the rejection were correct (mainly for Ana's method), due to not being able to achieve representativeness of the sample using the proposed procedure or the restriction of the study population. The justifications presented by the students in Ruiz-Reyes (2020) when identifying biased sampling

Table 5. Frequency and percentage of estimated mean value in a new sample (item 3, part a)

	Estimated value for the mean	Frequency	Percentage (%)
Correct	2.5	7	6.7
	Close to 2.5	46	43.8
	Interval	7	6.7
Incorrect	Different from 2.5	28	26.7
	Impossible to provide an answer	14	13.3
	No response	3	2.8
Total		105	100

Table 6. Options selected for the population mean in part b of the item 3

Selected options	Frequency	Percentage (%)
2.5 (value obtained)	21	20.0
We cannot be sure	53	50.5
I would take a larger sample	63	60.0
Between 2.4 & 2.6	24	22.9
No response	2	1.9

methods also differ from our results because only 10.2% of her students gave correct arguments and 42.8% partially correct in a similar situation to Luis' method. Likewise, 43.8% of the arguments were correct, and 12.4% were partially correct in a similar situation to Ana's method. In summary, our participants demonstrated better knowledge of sampling methods than students in Ruiz-Reyes (2020) and more argumentative capacity.

RESULTS IN ADVANCED CONTENT KNOWLEDGE

Below, we describe the results for item 3 and item 4, which assess this knowledge.

Relating the Population and Sample Mean

Table 5 presents the results of section a) of item 3, i.e., the estimated mean value for a new sample. The percentage of correct answers was high (57.2%), either by giving a value of 2.5, a value close to it, or a range of values that included 2.5.

Among incorrect answers were those who thought the new mean value would be different but did not indicate its value or those who suggested that it was impossible to answer. In this item, the percentage of non-responses was much lower than that in item 1. These results coincide with de Vetten et al. (2019), who found that prospective primary school teachers did not realize the possibility of generalizing the data in the sample to a population. Similarly to Batanero et al. (2022), some prospective secondary school teachers forgot a model previously obtained (the proportion of marked fish in the population) to predict results for a new sample. These findings imply the need to reinforce teachers' understanding of mathematical models and their use in prediction.

Table 7. Frequency (F) and percentage (P) of the estimated values for the population proportion (item 4, part a)

Estimated value of the population proportion	F	P (%)
Correct (10%)	27	25.7
Incorrect with proportional reasoning (25%, 40%, 47%, & 50%)	37	35.2
Incorrect with additive reasoning	10	9.5
Confuse	4	3.8
Impossible to provide an answer	4	3.8
No response	23	21.9
Total	105	100

Table 6 shows the percentage of participants who considered correct each of the options provided in section b of item 2 relative to the possible values of the population mean. The sum of the percentages is greater than 100 because many participants selected several options. Correct option 1 and option 4 were chosen by a minority of participants, whereas incorrect answers were the most chosen. These results indicate poor prospective primary school teachers' knowledge of the relationship between the sample and the population as well as the sampling properties to estimate from the sample mean.

Relating the Population and Sample Proportion in a Finite Population

Item 4 analyzes the estimated proportion of the population based on the sample proportion. In the correct responses, the participants computed the proportion of marked fish in the sample ($p = 0.1$) and assumed that the proportion in the population would be the same or close to this value. **Table 7** presents the estimated values of the population proportion (part a) with only one-fourth of correct answers. Again, the results demonstrated that prospective secondary school teachers' knowledge of the relationship between the sample and the population proportion was limited.

Some participants provided incorrect responses using proportional reasoning and by combining different data for the item. For example, they estimated the total number of fish as the sum of the two samples (450) and the sampling proportion as the quotient 225/450 = 50%, considering that the marked fish were the 25 marked in the first sample plus the 200 added to the pond on the first day. Another common response was to consider the total number of fish as the sum of the

Table 8. Frequency and percentage of the estimated values for the population (item 4, part b)

	Estimated value	Frequency	Percentage (%)
Correct	2,000	16	15.2
Incorrect with proportional reasoning		4	3.8
Incorrect with additive reasoning	425, 450, & 500	41	39.0
Other incorrect responses		17	16.2
Impossible to provide an answer		2	1.9
No response		25	23.8
Total		105	100

two samples, eliminating the number of fish marked in the first sample and obtaining for the sample proportion 200/425 = 47 (47%) and a total number of fish equal to 425. This response was also mentioned in Gómez-Torres et al. (2018), who also worked with prospective primary school teachers. Other participants estimated 25% of the marked fish in the pond, possibly because 25 was the number of fish marked in the second sample. Moreover, a few participants used additive reasoning in part a of item 4, and there were also confusing responses because some participants suggested that it was impossible to reply, while others did not answer.

Although the percentage of students who applied proportional reasoning in their answers was considerable (61%), there were gaps due to confusion of concepts (sample and population) or the identification of the data in the item. Furthermore, some participants (9.5%) used additive reasoning to establish their estimates. We also note that 21.9% of the prospective teachers did not respond to this question. The results were worse than those obtained by Ruiz-Reyes (2020) with students in the fourth year of secondary school, where she reported 51.1% of correct answers. However, the results were similar to those reported by Gómez-Torres et al. (2018) with prospective primary school teachers (20,4% correct responses); thus, it seems that prospective primary school teachers' knowledge to estimate the population proportion in a finite population from sampling was poorer than that of secondary school students.

Table 8 presents the results for the estimated value of the total number of fish in the pond (part b), obtaining a very small percentage of correct answers (15.2%), which is even worse than those reported by Gómez-Torres et al. (2018) also with prospective primary school teachers (20.4%). This was partly due to the fact that the participants carried over the errors made in part a. About half of the participants answered correctly using the population proportion to estimate (part b) that the total number of fish was 2,000 (as there were 200 marked fish in the pond).

A few incorrect responses were obtained by incorrect proportional reasoning in part b. Moreover, some participants used additive reasoning in part a, whereas it was mainly used in part b. Thus, some participants indicated 425, 450, or 500 as the total number of fish by

Table 9. Frequency (F) and percentage (P) of the estimated values for the number of marked fishes in a new sample (item 4, part c)

Estimated number of marked fish	F	P (%)
Correct 10	35	33.3
Incorrect 35	7	6.7
Incorrect much lower than 10	2	1.9
Incorrect much higher than 10	17	16.2
Higher than 100	5	4.8
Impossible to provide an answer	3	2.9
Other responses	11	10.5
No response	25	23.8
Total	105	100

misapplying proportional reasoning and combining different data.

The results were worse than those reported by Ruiz-Reyes (2020). This is partly because participants carried over errors made in part a. Again, although many studies have used proportional reasoning, confusion between the sample and the population, as well as incorrect identification of item data, leads to incorrect values. Furthermore, 46.7% of respondents used additive reasoning, and a few believed that the problem could not be solved. Finally, 23.8% of respondents did not answer this question.

Table 9 presents the results for part c, i.e., the number of fish marked in a new sample. In part c, the percentage of correct answers increased slightly (33.3%), but participants' responses were worse than those reported by Ruiz-Reyes (2020) with high school students. In the correct responses, our participants expected a value close to 10, suggesting good reasoning about the relationship between the sample and the population proportion. Among incorrect answers, those that overestimate the variability of the sample by providing values that are much smaller or larger than the correct values stand out. These results replicate the findings of previous research with prospective teachers, which showed that they faced some difficulties when reasoning about sample variability (Abu-Ghalyoun, 2021). In particular, Gómez-Torres et al. (2018) reported 54.1% correct answers in part c.

Additionally, a few participants provided values greater than the sample size or indicated that it was impossible to answer the question. The high percentage of non-responses is striking, exceeding the 17.2%

Table 10. Participants' understanding levels in SOLO taxonomy on sampling concepts

Level	Concept			
	Sample and population		Sampling variability	
	F	P (%)	F	P (%)
Pre-structural	15	14.3		
Uni-structural	10	9.5	26	24.8
Multi-structural	36	34.3	47	44.7
Relational	16	15.2	24	22.9
Imprecise	19	18.1	6	5.8
Do not reply	9	8.6	2	1.8

Note. F: Frequency & P: Percentage

reported by Gómez-Torres et al. (2018). Consequently, these prospective teachers did not use the population proportion model previously constructed in part a and part b to solve the new question in a similar way as the participants in the study by Batanero et al. (2022).

SAMPLING REASONING LEVEL

Finally, **Table 10** presents the distribution of understanding levels, following the SOLO taxonomy (Biggs & Collis 1982) summarized in **Table 1**, of the ideas of sample and population (adapted by Watson and Moritz, 2000) and sampling variability (adapted by Ko, 2016).

For the ideas of population and sample, an important part of prospective teachers did not reply, their responses were imprecise, or they were located at the pre-structural level, where the sample and population are confused, or a colloquial meaning of the concept is used, such as, for example, P55. The most frequent is the multi-structural level, where more than one aspect of the population and sample is correctly cited without relating them, as occurs in P17. The remaining participants were divided at the uni-structural level, at which a single aspect of the sample is cited (P14), and the relational level, at which several related aspects are described (P16). When compared with previous research, Watson and Moritz (2000) found that most students were located at the uni-structural level, advancing their level throughout the school year.

> P55: It sounds somewhat familiar to me. Sample: it is something that is taught as proof, as well as for a purchase, a problem, etc. Population: A group of people that surrounds a town, city, country, …

> P17: In a study, the population is the people to whom the study is directed; however, the sample is the part of the population from which the data is taken to analyze.

> P14: The sample and the population are concepts used in mathematics in the field of statistics. The population would be the whole, and the sample would be a part of the population that we use as the result of the entire population.

> P16: The "population" is the set of total people from whom we choose the "sample", which is the set of people to whom we carry out some type of research to obtain relevant data from scientific research.

Regarding the understanding of sample variability, most prospective teachers were classified at the multi-structural level in which the representativeness and variability of sampling are recognized; however, the relationship between the size of the sample and its sample variability is not understood (an example is P61, which selects options 1, 2, and 3). The remaining teachers were located at similar proportions at uni-structural and relational levels. In the first instance, prospective primary teachers acknowledged the sampling variability but indicated that it was not possible to make assumptions about the population (such as P48 selecting option 2 and option 3). At the relational level, we recognized sample variability and representativeness and considered sample size in the statistician's approximation of the population (P14, which selects option 1 and option 3). This recognition was achieved by a few participants but was not successful in Begué et al. (2023) with high school students and prospective teachers.

> P61: It is a sample with that solution; however, if the sample changes, the result changes. We cannot guarantee the exact value because the sample varies. The larger the sample, the greater the accuracy.

> P48: There are millions of families, and with only 1,000, the average is very random. We should use the entire population.

> P14: I think the first answer is correct since the approximate number is 2.5 because it is the result of the initial sample, and it was carried out on a good part of the population. The second is incorrect because the average number of diverse families in Spain is always approximately 2.5. As for the third answer, I think it is correct because the larger the sample, the more accurate is the average. The fourth response may be correct because the averages of 2.4 and 2.6 are still 2.5.

CONCLUSIONS

The study results indicate a reasonable sampling knowledge of common content in the participating sample. More specifically, the results obtained are, generally, better than those reported by research on secondary and high school students (Ruiz-Reyes, 2020) and high school (Watson & Kelly, 2005), as well as in the last year of primary education (Meletiou-Mavrotheris & Paparistodemu, 2015). Thus, as is visible in **Table 3**, most participants provided correct definitions of the concepts of population and sample, although only half provided an adequate example. Given the importance of context in statistics (Bargagliotti et al., 2020), an implication of this result is that teacher educators must reinforce their search for significant sampling examples from the media when preparing teachers (Batanero et al., 2019).

As discussed before and **Table 4** showed, we also observed competence in the detection of biased sampling methods, either by restricting the study population or selecting a different population. The arguments used were also generally consistent in selecting an acceptable sampling method. There was greater acceptance of random sampling than in previous studies with students, although there was agreement with them that stratified sampling was preferred to random sampling. Still, a part of the participants rejected random sampling, and a significant proportion was not able to realize that in one of the proposed sampling types, a population larger than the objective was taken (a center that could include high school students). Thus, discussing the sampling methods deeply is another important point that should be considered to improve teachers' knowledge.

Regarding advanced sampling knowledge, more than half of the participants provided correct answers for the estimated value of the mean for a new sample, but only a minority chose among several options the correct estimate of the population mean value. When estimating the proportion of the population from the sample, only one-fourth provided correct answers, whereas Ruiz-Reyes (2020) with high school students obtained 51.1%. The percentage of those who correctly calculated the size of a finite population, based on the sample proportion, was also very small, even lower than that obtained by Gómez-Torres et al. (2018) with other prospective primary school teachers. Estimating the proportion in a new sample was somewhat better. Consequently, it is necessary for prospective primary school teachers to reinforce their advanced sampling knowledge.

Regarding the level of reasoning at which the participants were located according to the SOLO taxonomy (Biggs & Collis 1982), the multi-structural level predominates, concerning the ideas of population and sample, as well as variability in sampling. In other words, different characteristics of these concepts were not related, which did not occur either in some of the prospective secondary school teachers in Begué et al. (2023).

In summary, despite the limitations of the study, given the small sample size and the use of few evaluation items, the research indicates points in which the training of primary school teachers must be reinforced to achieve success in the implementation of elementary ideas of inference and frequency probability in this educational cycle. These points include the sampling method, the relationship between the sample and the population mean, and the relationship between the sample and the population proportion. It is necessary to implement teaching actions based on simulation activities with manipulatives or technology where prospective teacher experiment sampling with progressively increasing sample sizes is conducted to become familiar with these ideas.

Of course, research must continue to design, experiment, and evaluate such teaching proposals, and also focus on the different facets of the teacher's didactic-mathematical knowledge, which were not considered in this study.

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APPENDIX A: QUESTIONNAIRE

Item 1

a. Have you ever heard the words: Sample and population? Explain the meaning of each of these words.
b. Write an example of a situation in which a sample must be used and indicate what the sample is and the population from which it is taken.

Item 2

In a secondary school, students conducted a survey to determine the number of books that students from the 1st to 4th grades of CSE read in a school year.

- María placed the names of the students from each course on the paper in a ballot box, randomly selecting 15 from each course.
- Luis asked the 60 students who were members of the school's reading club.
- Rocío placed papers with the names of all students at the secondary school in an urn and randomly selected 60 students.
- Ana asked all 1st grade students and obtained 70 responses.

1. Is María's procedure correct? Why or why not?
2. Is Luis' procedure correct? Why or why not?
3. Is Rocío's procedure correct? Why or why not?
4. Is Ana's procedure correct? Why or why not?

Item 3

The National Statistics Institute (NSI) conducted a survey of 1,000 Spanish families and found an average number of 2.5 members in those families.

a. If the NSI conducts another survey of 1,000 additional families next month, what value do you think they will get for the average number of family members? Why?
b. What value would you consider as the average number of family members of all Spanish families? (mark with a cross the correct answers):
   - ☐ 2.5 as it was the first value obtained by the NSI.
   - ☐ We cannot be sure of the value because the average number will change when other families are interviewed.
   - ☐ I would take a larger sample to calculate the mean.
   - ☐ I would guess it is between 2.4 and 2.6 because the NSI only interviewed a part of the population.

Write down the reasons that enabled you to identify correct or incorrect answers in the previous section.

Item 4

There are some fish in a pond, but the owner is unsure about how many there are. He took 200 fish, marked them, and returned to the pond. The next day, the owner took 250 fish from the pond and found that 25 of them were marked and the remainder were not:

a. What is the approximate proportion of marked fish in the pond?
b. What is the approximate number of fish in the pond?
c. If the owner takes 100 more fish from the pond, then approximately how many will be marked?