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# Praxeological analysis of linear algebra content presentation: A case study of Indonesian mathematics textbooks

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**Abstract**

Textbooks play a critical role in shaping students' understanding of algebra. This study aims to evaluate the representation of two-variable linear equation systems in grade 8 mathematics textbooks through a practical approach. Using praxeological analysis based on the anthropological theory of the didactic, the research examines tasks ($T$), techniques ($\tau$), technologies ($\theta$), and theories ($\Theta$) in textbook content. A reference epistemological model is employed to map the transition from one-variable to multi-variable linear equations. The findings reveal that while textbooks effectively introduce concepts and support procedural problem-solving, they often neglect deeper conceptual understanding. This gap may lead to students developing a limited procedural focus and encountering epistemological and didactic learning obstacles. The study concludes that textbooks should integrate not only techniques but also the underlying technology and theory to foster meaningful learning. The research contributes a framework for textbook analysis and offers recommendations to improve algebra content delivery in schools.

**Keywords:** anthropological theory of the didactic, didactic analysis, mathematics textbook, praxeological analysis, systems of linear equations

## INTRODUCTION

Textbooks serve as a primary resource in the teaching and learning process of mathematics (Pepin & Gueudet, 2020), especially in Indonesia. For students and teachers in schools, mathematics textbooks are an important guide to understanding mathematical concepts (Berisha & Bytyqi, 2020). Textbooks are equally essential for teachers in designing and delivering mathematics lessons to their students (Rezat et al., 2021; Ulusoy & İncikabı, 2020). Consequently, the quality of content presentation in textbooks significantly influences the learning process and its overall effectiveness (Kuncoro et al., 2024; Yunianta et al., 2023); if the material in the textbook is presented well, students can understand the concepts more easily (Hendriyanto et al., 2023).

Algebra, especially linear algebra, plays an important role in mathematics education (Fardian et al., 2025; Utami et al., 2024). Linear algebra not only serves as a foundation for studying advanced mathematics but also has many applications in everyday life (Ramírez-Montes et al., 2021; Spooner et al., 2024). The concept of linear algebra is often used in financial management, data analysis, and problem solving in various fields, including science and engineering (Veith et al., 2023). Therefore, the ability to understand and apply linear algebra concepts is important because it helps students think critically and analytically when facing real-world challenges.

Indonesia has long produced its own mathematics textbooks for use in schools. To date, there are more than one series of textbooks that can be used in class.

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**Contribution to the literature**

- This study analyzes the representation of praxeological components in linear algebra content, focusing on systems of linear equations in Indonesian textbooks.
- This study identifies gaps between procedural techniques and conceptual understanding, highlighting potential learning obstacles in current textbook designs.
- This study provides a foundation for improving textbook content and supports further research on enhancing algebra instruction through balanced didactic approaches.

Textbooks are free to use and schools ensure that each student receives relevant materials. Teachers' forums at the district level will decide which texts to use, and this decision is made collectively. Mathematics learning with textbooks includes activities such as reading explanatory texts and new materials, examining tables and graphs, observing solved problems, and doing exercises (Hendriyanto et al., 2023; Utami et al., 2024). During the learning process, teachers can integrate the use of textbooks that students utilize for studying. This allows a specific mathematics textbook to be employed as diverse learning materials in each mathematics class. Teachers can use the textbook during lessons as a source of practice problems or fully optimize all the materials presented in the textbook (Kuncoro et al., 2024).

The analysis of mathematics textbooks has been extensively explored in several countries, focusing on various dimensions of textbook content and pedagogy. Ho Cheong (2024) analyzed the requirements of textbook exercises using a five-dimensional approach, while Li and Wang (2024) examined how textbooks provide opportunities for learning and their correlation with students' achievements. In the Indonesian context, Utami et al. (2024) highlighted the urgency of integrating higher-order thinking skills into mathematics textbooks. Similarly, Yang and Sianturi (2022) compared algebraic problems across several countries, including Indonesia, showcasing different approaches to presenting mathematical concepts.

Despite these efforts, research in Indonesia still largely focuses on curriculum alignment and visual presentation aspects of textbooks (Rahmawati et al., 2020; Wijaya et al., 2022). Studies employing praxeological analysis or the anthropological theory of the didactic (ATD) are scarce. The praxeological framework, implemented in various countries, has proven valuable in analyzing how textbooks organize knowledge and facilitate the learning process. Takeuchi and Shinno (2020) applied praxeological analysis to compare geometry topics in Japanese and English textbooks, and dos Santos Verbisck and Bittar (2021) explored probability teaching in Brazilian textbooks. These studies underline the potential of ATD to reveal deeper insights into how mathematical knowledge is structured and conveyed. Kuncoro et al. (2024) investigated the use of praxeological analysis in Indonesian and Singaporean textbooks, particularly in understanding geometrical similarity, highlighting cross-national differences in the treatment of mathematical concepts.

In Indonesia, textbooks often emphasize problem-solving techniques without providing adequate conceptual explanations (Hendriyanto et al., 2023; Kuncoro et al., 2024). ATD has gained attention as an approach that can help understand how didactic processes occur in mathematics learning in educational settings (Chevallard et al., 2015). According to ATD theory, didactic understanding plays an important role in translating educators' academic knowledge into knowledge taught in schools. This process causes content reduction in the mathematics teaching and learning process (Bosch et al., 2021; Chevallard, 2006). Research by applying the ATD framework in algebra found that the way teachers utilize materials in textbooks has a significant effect on students' learning speed (Daher et al., 2022; Godino et al., 2019).

In the context of linear algebra, several studies have examined the effectiveness of different teaching approaches and presentations of mathematics textbooks. Research has examined students' ability to make mathematical connections when solving systems of linear equations, showing that students frequently face difficulties in linking various algebraic representations to find solutions (Hidayati et al., 2020). In the same way, the manner in which textbooks present systems of three-variable linear equations has a significant impact on students' mathematical communication skills (Mastuti et al., 2022). These findings emphasize the need for careful and deliberate design of algebraic tasks in textbooks to enhance both procedural skills and a deeper conceptual understanding of linear equations.

Research trends regarding the use of textbooks in mathematics education, particularly in algebraic concepts, indicate that textbooks continue to be a vital resource for students in building their algebraic knowledge (Fardian et al., 2024; Utami et al., 2022). Although textbooks are important resources for teachers and students that can help, they can also be a hindrance to students' understanding of mathematical concepts (Pepin & Gueudet, 2020; Rezat et al., 2021). Analysis of textbooks revealed that many tasks are not designed to elicit deep conceptual understanding, but rather to apply procedures that are more akin to computational algebra (Polat & Dede, 2023).

Praxeological analysis in the ATD framework is useful for evaluating the way algebraic concepts are presented in mathematics textbooks. This approach refers to the types of tasks that students work on and the strategies and reasons they use to complete them (Chevallard & Bosch, 2020a). Studies indicate that praxeological analysis offers valuable insights into how teaching materials and resources can be utilized to enhance the quality of algebra instruction in the classroom (Utami et al., 2022; Wijayanti & Winslow, 2017). By adopting the ATD framework, particularly its praxeological components (tasks, techniques, technologies, and theories), this study addresses the gap by analyzing grade 8 mathematics textbooks, focusing on the topic of two-variable linear equation systems. Based on aspects related to algebra learning, especially linear equations, it is necessary to conduct further research on how textbooks improve or hinder students' understanding of these concepts. The aim of this study is to examine how linear equations are presented in 8th grade mathematics textbooks and assess the effectiveness of the approaches used in the textbooks to support mathematics learning.

Weaknesses in the delivery of material often cause students to solve problems procedurally without gaining a deep understanding of basic concepts such as linear algebra (Grugeon-Allys & Pilet, 2024). This results in the need to take a more comprehensive approach in analyzing the structure and presentation of textbook content. This is important because it can help students develop a deeper understanding than following procedural steps. With a better understanding, it is hoped that students will be able to apply the concept of linear algebra in various contexts in everyday life.

This study aims to provide new insights into how the praxeological components $(T, \tau, \theta, \Theta)$ are represented in mathematics textbooks, especially in the field of linear algebra. The findings of this study are expected to offer valuable insights to teachers and textbook developers, enabling them to enhance the quality of mathematical presentation and learning for students. This research makes a significant contribution to the process of understanding and teaching linear algebra at the secondary school level. The focus of this study is how textbooks present types of tasks ($T$), techniques ($\tau$), technologies ($\theta$), and theories ($\Theta$). How the balance between these components affects students' conceptual understanding during the assessment of linear algebra learning. This study is expected to be a starting point for improving the presentation of mathematics textbooks in Indonesia and improving students' learning experiences.

## METHOD

This study employs a praxeological framework based on the ATD, developed by Chevallard (2006), to analyze mathematics textbooks for junior high schools in Indonesia. ATD views every human action, including teaching mathematics, as a series of tasks to be completed, there are techniques used, and reasons why the techniques are used (Chevallard & Bosch, 2020b; Gascón, 2024). Mathematics textbooks are ideal objects to be analyzed using this approach, because they are the product of collaboration between individuals (authors) and institutions (Ministry of Education, Culture, Research and Technology Development). Praxeological analysis allows for the identification of the relationship between institutional praxeology (schools) and individual praxeology (textbook authors), offering a deeper understanding of how mathematical knowledge is presented in textbooks and how it is taught in the classroom (Rezat, 2024). The study proposes the reference epistemological model (REM) as the basis for interpreting mathematical knowledge (Benito & da Silva, 2021), particularly in the transition from linear equation in one variable to system of linear equations in $n$ variables, $n \geq 4$.

This analysis is also supported by the theory of learning obstacles in didactic situations proposed by Brousseau (2006). This theory highlights three key elements of the learning process, namely student activity, required knowledge, and teacher instruction. In the context of textbooks, these elements are reflected in the tasks presented, the techniques taught and the reasons behind these techniques are given (Wijayanti & Winslow, 2017). This study combines praxeological analysis and the theory of learning obstacles to identify challenges in how mathematical knowledge is presented in school textbooks and to offer relevant suggestions for improvement.

### Research Object

This study examines a mathematics textbook on linear algebra for eighth-grade high school students, which is one of two books (teacher's book and student's book) produced by the Indonesian Ministry of Education, Culture, Research, and Technology (Kemendikbudristek) and used in the 2024 independent school curriculum. The focus of this study is on the student's book (Tosho et al., 2021). This textbook is utilized by teachers for teaching high school mathematics, particularly algebra. The selection of this textbook is based on its algebra content and its role as the primary learning resource for students in the classroom.

### Data Collection Process

The research data were gathered through a praxeological analysis of the content in the selected textbooks. The various types of tasks and techniques outlined in the mathematics textbooks were then explained and assessed using the praxeological components. The data were subsequently analyzed qualitatively to examine the presentation of task types ($T$), techniques ($\tau$), technology ($\theta$), and theories ($\Theta$), along with the relationships between the tasks.

## Data Analysis

The data analysis process in this study involves two key steps: praxeological analysis and analysis of learning barriers. All of these steps include several steps to ensure that the data is analyzed thoroughly based on the framework used.

### *Stage 1. Reference epistemological model*

REM are theoretical constructs used to interpret and analyze the teaching and learning of mathematical content. They address critical questions such as how mathematical concepts are understood within a specific educational institution, their relevance, and how they connect to other mathematical domains. REM, as proposed by Bosch and Gascón (2005), are built from empirical data and focus on the praxeological frameworks that guide mathematical learning. These models are dynamic and provisional, constantly refined through research and practice to ensure their alignment with the evolving nature of mathematical education.

In the context of teaching linear equations in two variables, the REM helps articulate the transition from elementary to secondary education. It connects institutionalized knowledge in primary education with more advanced mathematical concepts in secondary education, ensuring continuity in learning (Benito & da Silva, 2021). By emphasizing practical problems and the evolving nature of mathematical inquiry, the REM provides a structured pathway for students to deepen their understanding and solve increasingly complex problems (Sahara et al., 2025).

### *Stage 2. Praxeological analysis*

Praxeology is the core of the ATD, where Chevallard (2006) states that praxeology is a theory of in-depth analysis of human action and behavior. This concept states that there is no human action or behavior without a reason underlying it (Chevallard et al., 2015). Praxeology is composed of two interconnected elements: praxis and logos. Praxis (the practical component) pertains to human activity, whereas logos (the knowledge component) relates to the reasoning or thought processes that underpin the action (Chevallard, 2006). In mathematics education, praxeology can serve as a framework for designing a sequence of tasks for students, aiding them in acquiring knowledge about specific mathematical concepts. **Figure 1** illustrates the role of the four components of praxeology.

The first step in praxeological analysis is to select high school mathematics textbooks containing algebraic content to be analyzed. The analysis focuses on identifying the praxeological components, namely tasks

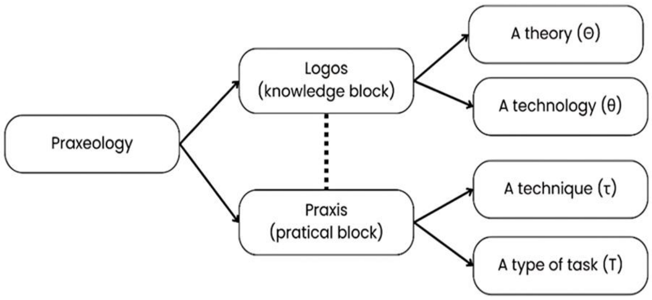



**Figure 1.** Four components of praxeology (Adapted from Chevallard, 2006)

($T$), techniques ($\tau$), technologies ($\theta$), and theories ($\Theta$) contained in mathematics textbooks (Bosch et al., 2017; Pansell, 2023). The next step is then to classify the mathematics tasks into a praxeological table. The identified tasks are analyzed based on the techniques used to solve them, the justification of the technology that supports the technique, and the underlying theory (Takeuchi & Shinno, 2020). The praxeological table helps researchers identify and group the tasks presented, revealing their techniques, technologies and theories (Gök & Erdoğan, 2023).

Each identified element $(T, \tau, \theta, \Theta)$ is then coded according to the ATD theoretical framework. This coding aims to facilitate categorizing and analyzing the presentation of linear algebra tasks in mathematics textbooks. Furthermore, researchers conduct theoretical triangulation, where the results of coding and praxeological analysis are verified using the basic concepts of ATD and several relevant literatures. This triangulation was carried out to ensure that each result was in line with Chevallard's (2006) praxeological framework, so that the results of the analysis could be relied upon.

## Data Validity

To improve the validity of the collected data, the principle of reliability between observers of data correspondence was applied. This step involves several independent reviewers who examine the data in an effort to confirm the results and conclusions. The process of drawing conclusions is also directed by the principle of coherence, ensuring that the analysis results align consistently with the theory, particularly the ATD theory and Brousseau's (2006) didactic theory.

By using praxeological analysis and learning barrier analysis, it not only reveals the praxeological structure of the selected mathematics textbooks, but how algebraic content is also presented, and learning obstacles that may arise when students learn using the selected textbooks. The results of these two analyses can help evaluate the advantages and disadvantages of selected mathematics textbooks and provide suggestions for improving the presentation of algebraic material in the future.

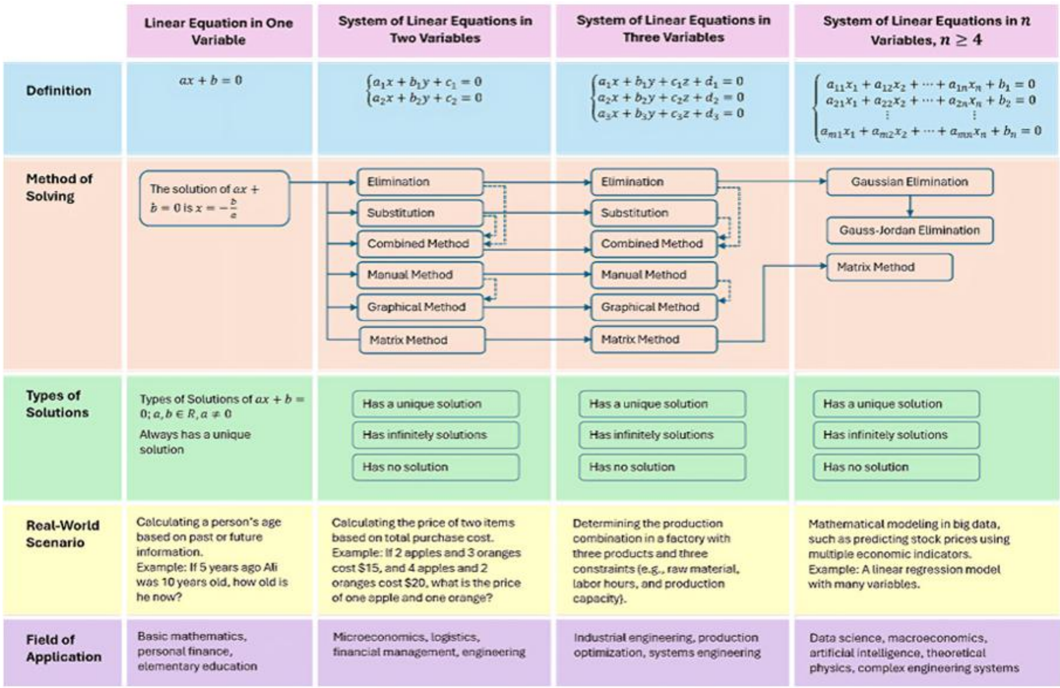



**Figure 2.** The REM for linear equation in two variables (Source: Authors' own elaboration)

## RESULTS

The findings of this study show that using the praxeological reference model to analyze mathematics textbooks offers a more organized perspective on the presentation of content, especially in the system of linear equations with two and three variables. Based on the analysis type of tasks ($T$), techniques ($\tau$), technologies ($\theta$), and theories ($\Theta$) presented in the textbooks, it was found that although the techniques for solving systems of linear equations are delivered procedurally, there are still shortcomings in terms of conceptual understanding. Most of the tasks presented in the textbooks focus on applying procedural steps without providing in-depth explanations of the algebraic theory underlying these methods. This has the potential to become a learning obstacle in students' development of understanding more complex algebraic concepts.

### Reference Epistemological Model

#### *Logos*

Linear algebra is a field of mathematics that deals with vector spaces, linear transformations, and systems of linear equations, providing a fundamental basis for applications in areas like science, engineering, and economics (Rensaa et al., 2021). A key topic in this field is the study of linear equations in two variables, which introduces students to systems of equations and their real-world applications. A linear equation in two variables takes the form $ax + by + c = 0$, where $a, b, c$ are constants, and $x, y$ are variables. The graph of such an equation is a straight line on the Cartesian plane.

There are several methods to solve linear equations in two variables. These include the graphical method, where equations are plotted to find their intersection point; the substitution method, which substitutes one variable with its equivalent from another equation; and the elimination method, where equations are added or subtracted to eliminate one variable and solve for the other. The solutions to systems of linear equations can be categorized into three types: unique solutions, where the lines intersect at one point $\left(\frac{a_1}{a_2} \neq \frac{b_1}{b_2}\right)$; infinite solutions, where the lines coincide $\left(\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}\right)$; and no solution, where the lines are parallel $\left(\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}\right)$. These equations are applied in solving real-world problems, such as cost analysis, resource allocation, and modeling scenarios in fields like economics, physics, and engineering, as demonstrated in **Figure 2**.

#### *Praxis*

The praxis section in the REM for linear equations in two variables is designed to support the body of knowledge in linear algebra as it is introduced and developed across elementary to secondary education. This REM serves as a bridge that expands on the institutionalized knowledge at the primary level and guides its evolution into more advanced mathematical concepts at the secondary level. By connecting two consecutive levels of education, this REM facilitates a seamless transition in understanding and application, ensuring continuity in mathematical learning. The construction of this REM is rooted in research and literature from mathematics education. It integrates knowledge and techniques essential for developing students' competencies in solving systems of linear equations. Three primary techniques form the foundation of the praxis in this model.

**Perceptual ($\tau_1$):** This involves the ability to model real-world problems into systems of linear equations. Students are encouraged to interpret contextual problems and translate them into mathematical representations, helping them develop a concrete understanding of how linear equations apply to practical scenarios.

**Operational ($\tau_2$):** This technique emphasizes the use of basic arithmetic operations, such as addition, subtraction, multiplication, and division. These operations are fundamental in solving linear equations through methods like substitution or elimination. Mastery of these skills equips students with the tools needed to navigate increasingly complex problems.

**Instrumental ($\tau_3$):** Here, students learn to use tables as a tool to determine the dependent variable's value when the independent variable is given. This approach not only aids in visualization but also strengthens the connection between abstract mathematical concepts and their practical application.

The praxis framework is deliberately structured to reinforce students' understanding and skill development at each educational stage. For instance, students in elementary education focus on the perceptual aspect, gradually building the ability to model simple problems. As they progress to secondary education, operational and instrumental techniques are introduced and refined, enabling students to solve more sophisticated systems of equations effectively. This integrated approach ensures that students are not only prepared to handle the increasing complexity of linear equations but are also able to connect their mathematical knowledge to real-world applications. By developing these competencies, REM not only aligns with the curriculum but also promotes critical thinking and problem-solving skills, which are vital for academic achievement and continuous learning.

## Praxeological Analysis

### *Logos*

In the logos of the Praxeological Analysis for solving systems of linear equations in two variables, two primary theoretical approaches are identified based on selected mathematics textbooks. These approaches provide a structured framework for guiding students in understanding and solving problems related to the topic. However, a critical observation reveals that both the arithmetic (proportional) and algebraic methods are introduced only briefly and superficially in most textbooks. This limited exposure can create significant learning obstacles for students as they engage with the material.

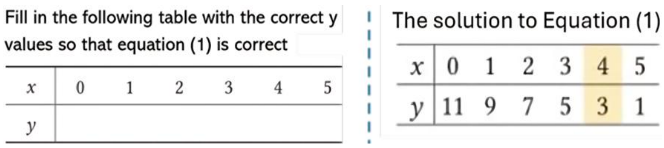



**Figure 3.** Arithmetic (proportional) method (Tosho et al., 2021)

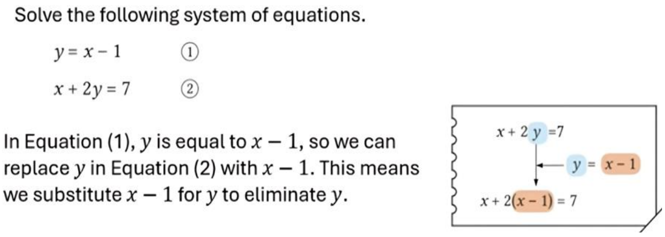



**Figure 4.** Algebraic method (Tosho et al., 2021)

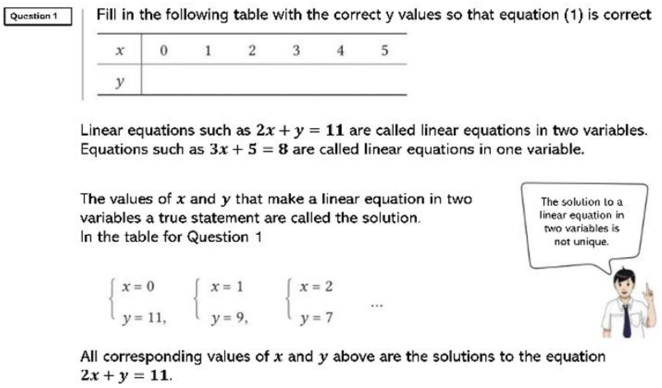
The arithmetic (proportional) method, for instance, emphasizes exploring relationships between variables through tabular representations, as shown in **Figure 3**. While this activity can help students grasp the concept of dependent and independent variables, its limited treatment in textbooks often fails to foster a deeper understanding of proportional reasoning or its broader applications.

The algebraic method, on the other hand, introduces formal structures of linear equations and problem-solving strategies such as substitution and elimination. Students learn to distinguish between equations in one variable, such as $3x + 5 = 8$, and those in two variables, such as $y = x - 1$ and $x + 2y = 7$, as shown in **Figure 4**. However, the superficial coverage in textbooks often does not provide enough practice or explanation, particularly in the transition from interpreting equations to selecting and applying appropriate techniques like substitution or elimination. Tasks are limited to standard forms with direct solutions, offering little variation or opportunity for students to reflect on the rationale behind each method. To better understand the nature of this learning obstacle, a praxeological analysis is carried out based on the REM, which serves as the foundation for identifying the expected types of tasks (praxis) and justifications (logos) in learning linear equations. This insufficient emphasis on both procedural techniques and their theoretical underpinnings can lead to learning obstacles, particularly as students' progress to more advanced topics that require a strong foundation in solving systems of linear equations.

When textbooks fail to present these methods comprehensively, students may struggle with recognizing patterns, connecting concepts, and applying algebraic techniques effectively. Thus, addressing this gap in textbook content is crucial. A more robust and detailed exploration of both arithmetic and algebraic methods in teaching materials could enhance students' understanding and reduce potential learning barriers.

**Table 1.** Types of tasks ($T$) in systems of linear equations in two variables

| $T$ | Description |
|---|---|
| $T_1$ | Given two linear equations in two variables, students are asked to find the solution to the system of equations. |
| $T_2$ | When presented with a system of two-variable linear equations, students are tasked with finding the common solution to both equations. |
| $T_3$ | Given a system of two-variable linear equations, students are asked to solve it using the elimination method. |
| $T_4$ | For a system of two-variable linear equations, students are instructed to utilize the substitution method to solve the variables. |
| $T_5$ | In real-life problem scenarios, students are asked to model the problems as a system of three-variable linear equations, where each equation contains two variables. |
| $T_6$ | When given a system of three-variable linear equations, students are asked to solve the system using either the elimination or substitution method. |



Question 1 Fill in the following table with the correct y values so that equation (1) is correct

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $y$ | | | | | | |

Linear equations such as $2x + y = 11$ are called linear equations in two variables. Equations such as $3x + 5 = 8$ are called linear equations in one variable.

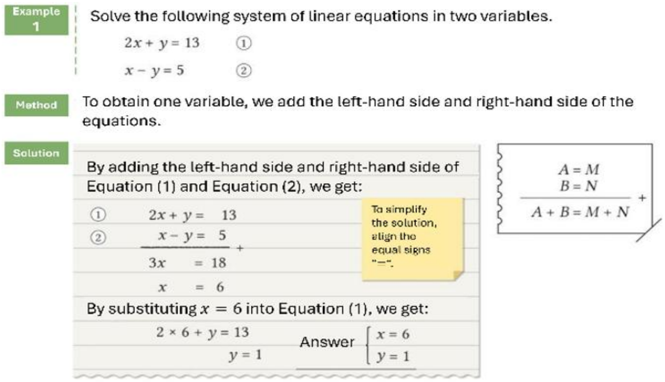
The values of $x$ and $y$ that make a linear equation in two variables a true statement are called the solution. In the table for Question 1

The solution to a linear equation in two variables is not unique.

$\begin{cases} x = 0 \\ y = 11, \end{cases}$ $\begin{cases} x = 1 \\ y = 9, \end{cases}$ $\begin{cases} x = 2 \\ y = 7 \end{cases}$ ...

All corresponding values of $x$ and $y$ above are the solutions to the equation $2x + y = 11$.

**Figure 5.** $T_{1.1}$ is an example of the type of task $T_1$ (Tosho et al., 2021)



Example 1 Solve the following system of linear equations in two variables.

$2x + y = 13$ ①

$x - y = 5$ ②

Method To obtain one variable, we add the left-hand side and right-hand side of the equations.

Solution By adding the left-hand side and right-hand side of Equation (1) and Equation (2), we get:

① $2x + y = 13$

② $x - y = 5$ +

$3x = 18$

$x = 6$

To simplify the solution, align the equal signs "=".

$A = M$, $B = N$, $A + B = M + N$

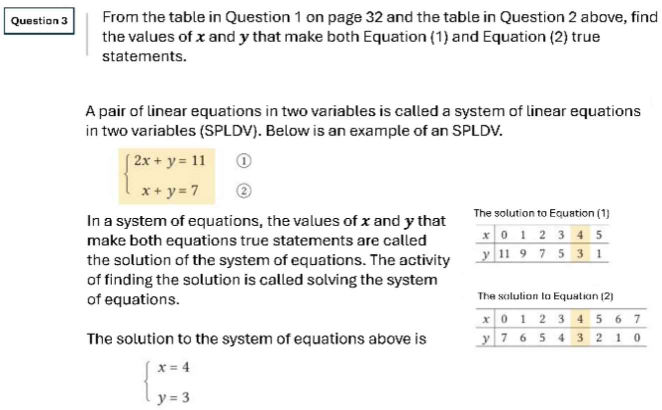
By substituting $x = 6$ into Equation (1), we get:

$2 \times 6 + y = 13$

$y = 1$

Answer $\begin{cases} x = 6 \\ y = 1 \end{cases}$

**Figure 7.** $T_{3.1}$ is an example of the type of task $T_3$ (Tosho et al., 2021)



Question 3 From the table in Question 1 on page 32 and the table in Question 2 above, find the values of $x$ and $y$ that make both Equation (1) and Equation (2) true statements.

A pair of linear equations in two variables is called a system of linear equations in two variables (SPLDV). Below is an example of an SPLDV.

$\begin{cases} 2x + y = 11 & ① \\ x + y = 7 & ② \end{cases}$

In a system of equations, the values of $x$ and $y$ that make both equations true statements are called the solution of the system of equations. The activity of finding the solution is called solving the system of equations.

The solution to Equation (1)

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $y$ | 11 | 9 | 7 | 5 | 3 | 1 |

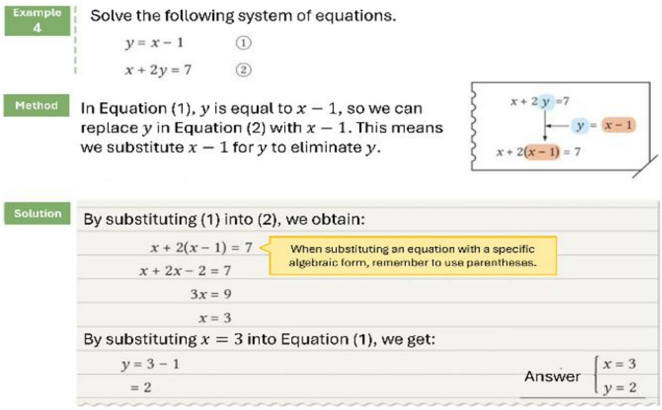
The solution to Equation (2)

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $y$ | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |

The solution to the system of equations above is

$\begin{cases} x = 4 \\ y = 3 \end{cases}$

**Figure 6.** $T_{2.1}$ is an example of the type of task $T_2$ (Tosho et al., 2021)



Example 4 Solve the following system of equations.

$y = x - 1$ ①

$x + 2y = 7$ ②

Method In Equation (1), $y$ is equal to $x - 1$, so we can replace $y$ in Equation (2) with $x - 1$. This means we substitute $x - 1$ for $y$ to eliminate $y$.

$x + 2y = 7$, $y = x - 1$, $x + 2(x - 1) = 7$

Solution By substituting (1) into (2), we obtain:

$x + 2(x - 1) = 7$

When substituting an equation with a specific algebraic form, remember to use parentheses.

$x + 2x - 2 = 7$

$3x = 9$

$x = 3$

By substituting $x = 3$ into Equation (1), we get:

$y = 3 - 1$

$= 2$

Answer $\begin{cases} x = 3 \\ y = 2 \end{cases}$

**Figure 8.** $T_{4.1}$ is an example of the type of task $T_4$ (Tosho et al., 2021)

By doing so, educators can ensure that students are better equipped to navigate the challenges of systems of linear equations and related mathematical concepts.

### *Praxis*

This study's praxeological analysis centers on the topic of systems of linear equations in two variables, as presented in an 8th grade junior high school mathematics textbook. The analysis explores the types of tasks, the methods used to solve them, and the technologies and the theories that support these methods (Pansell, 2023). **Table 1** presents the results of the praxeological analysis based on the types and number of tasks.

**Figure 5** displays an example type of task $T_1$, specifically $T_{1.1}$: determining the value of the dependent variable when the independent variable is known.

**Figure 6** shows type of task $T_2$, specifically $T_{2.1}$: determining the solution of a system of two-variable linear equations through steps at level 1 of $T_1$.

**Figure 7** presents a type of task $T_3$, specifically $T_{3.1}$: determining the solution of a system of two-variable linear equations using the elimination method.

**Figure 8** displays a type of task $T_4$, specifically $T_{4.1}$: determining the solution of a system of two-variable linear equations using the substitution method with various forms of linear equations.

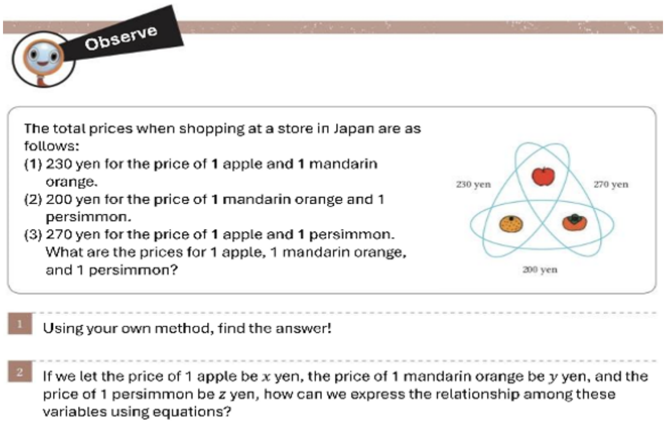



**Figure 9.** $T_{5.1}$ is an example of the type of task $T_5$ (Tosho et al., 2021)

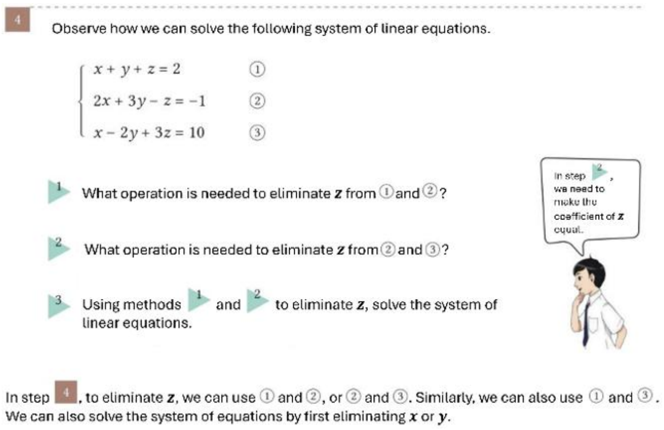



**Figure 10.** $T_{6.2}$ is an example of the type of task $T_6$ (Tosho et al., 2021)

**Table 2.** Identification of types of tasks

| No | Topic | Pages | Type of task sequence |
|---|---|---|---|
| 1 | Systems of equations and solutions | 32-33 | $T_1 \rightarrow T_2$ |
| 2 | Methods for solving systems of equations | 34-42 | $T_3 \rightarrow T_4$ |
| 3 | Development of systems of equations | 44-45 | $T_5 \rightarrow T_2 \rightarrow T_4 \rightarrow T_6$ |

**Table 3.** Task levels

| $T$ | Frequency of tasks (n = 76) | Level 1 ($T_n[1]$) | Level 2 ($T_n[2]$) | Level 3 ($T_n[3]$) |
|---|---|---|---|---|
| $T_1$ | 2 (2.63%) | 2 (100%) | 0 | 0 |
| $T_2$ | 4 (5.26%) | 4 (100%) | 0 | 0 |
| $T_3$ | 26 (34.21%) | 26 (100%) | 0 | 0 |
| $T_4$ | 31 (40.78%) | 16 (51.61%) | 15 (48.38%) | 0 |
| $T_5$ | 1 (1.31%) | 0 | 0 | 1 (100%) |
| $T_6$ | 12 (15.78%) | 0 | 0 | 12 (100%) |

**Figure 9** shows a type of task $T_5$, specifically $T_{5.1}$: modeling real-life problems into a system of three-variable linear equations.

**Figure 10** presents a type of task $T_6$, specifically $T_{6.2}$: determining the solution of a system of three-variable linear equations using the elimination or substitution method.

**Table 2** presents the classification of task types, which are categorized into three main topics: systems of equations and their solutions, methods for solving systems of equations, and the development of systems of equations. The sequence of task types within each topic is not necessarily linear but is organized to align with the learning progression required for students.

On pages 32-33, the types of task presented are $T_1 \rightarrow T_2$, which focus on the basic introduction of systems of equations and how to find their solutions. On pages 34-42, the topic "Methods for solving systems of equations" presents the sequence $T_3 \rightarrow T_4$, where students are taught the elimination and substitution methods to solve systems of two-variable linear equations. Meanwhile, on pages 44-45, in the topic "Development of systems of equations," a more varied sequence of tasks is presented, namely $T_5 \rightarrow T_2 \rightarrow T_4 \rightarrow T_6$, which combines various technique ($\tau$) previously learned to solve more complex problems. The sequence of tasks is structured to ensure that students first grasp the fundamental concepts and then progress to applying more advanced techniques. The non-linear sequence allows students to gradually build skills, reinforcing their understanding before moving on to more advanced and challenging applications.

**Table 3** presents the task levels categorized by the complexity involved in solving systems of linear equations with two variables and systems of linear equations with three variables. Each type of task is categorized into three levels: level 1 includes basic tasks, level 2 introduces variations in equation forms, and level 3 includes more complex tasks involving three variables. In task $T_1$, all problems are at level 1, which requires finding the value of the dependent variable given the value of the independent variable. Similarly, task $T_2$ is entirely at level 1, where students are required to find the solution of a system of linear equations with two variables through the solution steps learned in $T_1$. Task $T_3$, related to the elimination method, is also fully at level 1, concentrating on solving two-variable linear equation systems using this method.

Unlike the previous tasks, task $T_4$ is divided into two levels. About 51.61% of $T_4$ problems are at level 1, requiring students to solve systems of linear equations with two variables using the substitution method.

**Table 4.** Praxeological analysis results

| $T$ | Technique ($\tau$) | Technology ($\theta$) | Theory ($\Theta$) |
|---|---|---|---|
| $T_1$ | $\tau_1$: The ability to model everyday problems into systems of linear equations (perceptual)<br>$\tau_2$: Using arithmetic operations such as addition, subtraction, multiplication and division (operational)<br>$\tau_3$: Using a table to determine the value of the dependent variable if the independent variable is known (instrumental) | $\theta_1$: Determining the solution of a linear equations with two variables. | $\Theta_1$: Systems of linear equations with two variables. |
| $T_2$ | $\tau_1$: The ability to model everyday problems into systems of linear equations (perceptual)<br>$\tau_2$: Using arithmetic operations such as addition, subtraction, multiplication and division (operational)<br>$\tau_3$: Using a table to determine the value of the dependent variable if the independent variable is known (instrumental) | | |
| $T_3$ | $\tau_2$: Using arithmetic operations such as addition, subtraction, multiplication and division (operational) | $\theta_2$: Determining the solution of a system of linear equations with two variables. | |
| $T_4$ | $\tau_2$: Using arithmetic operations such as addition, subtraction, multiplication and division (operational) | | |
| $T_5$ | $\tau_1$: The ability to model everyday problems into systems of linear equations (Perceptual)<br>$\tau_2$: Using arithmetic operations such as addition, subtraction, multiplication and division (operational) | | |
| $T_6$ | $\tau_2$: Using arithmetic operations such as addition, subtraction, multiplication and division (operational) | $\theta_3$: Applying systems of linear equations with three variables. | $\Theta_2$: Systems of linear equations with three variables. |

Meanwhile, 48.38% of $T_4$ problems are at level 2, introducing variations in the form of linear equations in the substitution method. Task $T_5$ is only found at level 3, where students must model real-world problems into systems of linear equations with three variables. Finally, task $T_6$ is also entirely at level 3, concentrating on solving systems of three-variable linear equations through the elimination or substitution methods. The majority of tasks are at level 1, consisting of basic procedural tasks. Meanwhile, only $T_4$ demonstrates variation at level 2, and more complex tasks involving three variables ($T_5$ and $T_6$) are all at level 3, reflecting a higher conceptual challenge in modeling and solving systems of equations with more than two variables.

The techniques used in solving tasks related to systems of linear equations with two variables involve various arithmetic processes and basic algorithms, categorized into three types. Perceptual technique ($\tau_1$) relies on modeling real-world problems into mathematical form, for example, converting everyday problems into systems of linear equations. Operational technique ($\tau_2$) entails applying fundamental arithmetic operations like addition, subtraction, multiplication, and division to solve equations. Meanwhile, instrumental technique ($\tau_3$) uses aids like tables or graphs to facilitate equation solving and provide clearer visualization for students.

**Table 4** provides a comprehensive praxeological analysis, detailing each task associated with both the two-variable and three-variable linear equation systems is explained according to the technique ($\tau$), technology ($\theta$) and theory ($\Theta$) applied. $T_1$ aims to determine the solution of the two-variable linear equation system, where three techniques are used, namely perception ($\tau_1$), operational ($\tau_2$) and instrumental ($\tau_3$). The technology that supports the technique ($\tau_{1,2,3}$) is ($\theta_1$) the procedure used to find the solution of the linear system, supported by the basic theory of the two-variable linear equation system ($\Theta_1$). $T_2$ seeks a general solution to two two-variable linear equations, using the technique ($\tau_{1,2,3}$). The technology and theory behind the technique are also the same, namely ($\theta_1, \Theta_1$). For $T_3$, only the operational technique ($\tau_2$) is used to solve the two-variable linear equation system using the elimination method. The supporting technology ($\theta_2$) focuses on determining the system solution, with the same theory, namely the theory of the two-variable linear equation system ($\Theta_1$). $T_4$ involves the substitution method to solve the two-variable linear equation system, also using the operational technique ($\tau_2$). The supporting technology ($\theta_2$) is similar to $T_3$, with the theory still based on the basic theory of the two-variable linear equation system ($\Theta_1$). $T_5$ is more complex than other types of assignments because students must model real-world problems in a three-variable linear equation system. The techniques used in $T_5$ are perceptual and operational ($\tau_{1,2}$). The technology behind $T_5$ still focuses on solving the two-variable linear equation system ($\theta_2$), although this assignment begins to enter the domain of the three-variable system. The theory used also refers to the basic theory of the two-variable linear equation system ($\Theta_1$). As for $T_6$, students are asked to solve the three-variable linear equation system using the elimination or substitution method. The operational technique ($\tau_2$) is applied, while the supporting technology ($\theta_3$) helps apply the three-variable system solution method. The underlying theory of $T_6$ is the theory of systems of linear equations in three variables ($\Theta_2$) which is relevant to the complexity of linear equations.

## DISCUSSION

Mathematics textbooks for junior high school students, specifically covering systems of linear equations in two variables, exhibit an organized and systematic structure in presenting the material. The methodical presentation, especially through the elimination and substitution methods, is designed to help students build a strong arithmetic foundation. However, from a deeper didactic perspective, there are strengths and weaknesses in this material's presentation that merit further analysis.

### Organization of Tasks in the Textbook

The structure of how systems of linear equations in two variables are presented in the mathematics textbook shows a strong emphasis on teaching procedural techniques. Based on **Table 3**, 40.78% of tasks focus on the substitution method, while 34.21% focus on the elimination method. These two methods are introduced sequentially, giving students the opportunity to understand the basic procedures for solving systems of linear equations in two variables. This approach supports students' technical skills in applying mathematical operations to solve problems, which is crucial for preparing them to tackle more complex questions (Szabo et al., 2020). Organizing tasks in this technique-heavy manner is intended to ensure that students acquire the necessary arithmetic skills to solve linear equations.

However, this highly mechanistic approach has its drawbacks. Students who focus solely on the solution techniques, such as elimination and substitution, may encounter epistemological learning obstacles. This occurs because students may be able to solve equations procedurally, but they do not deeply understand the underlying theory of these techniques (Cabuquin & Abocejo, 2024). For example, the relationship between independent and dependent variables, which is vital for understanding a system of equations, is often overlooked in the teaching of procedural techniques. As a result, students focus only on how to solve the problem, without understanding why the technique is used or how the concept relates to real-life situations, such as the mathematical modeling of a problem.

When the textbook introduces systems of linear equations in three variables, there is a significant imbalance in task organization. Based on **Table 3**, only 1.31% of tasks focus on modeling real-world cases with three variables, indicating that students are not adequately guided in the transition from systems of linear equations in two variables to systems in three variables. In the context of algebra, understanding systems of linear equations in three variables requires deeper conceptual reasoning, as students must handle more variables and understand more complex relationships (Mastuti et al., 2022). The lack of exercises and assignments focused on three-variable linear equation systems results in students having little opportunity to adapt to the material. This can also make it difficult for students when faced with tasks that require a deeper understanding of three-variable linear equation systems (Henriques & Martins, 2022).

The limited presentation of three-variable linear equation systems in selected textbooks not only creates didactic obstacles but can also eliminate students' opportunities to develop a deeper understanding of algebraic concepts. The transition from two-variable to three-variable equations in algebra is not as simple as adding variables but also involves a level of complexity in the relationships between variables and how to find solutions to these equations. Applications and modeling of real-world problems involving three variables should provide a foundation for students to understand how three-variable linear equation systems work (Fardian et al., 2025). However, this issue is not adequately addressed in the selected textbooks. As a result, students can master the techniques for solving two-variable linear equation systems, but they are not conceptually ready to understand and apply these techniques in more complex systems, in this case three-variable linear equation systems.

To address this issue, mathematics textbooks should provide more varied transition tasks that certainly involve three-variable linear equation systems and provide more real-world contexts that can be related to algebraic concepts. Students should not only master the techniques of elimination and substitution but also understand how these techniques can be applied to solve problems involving multiple variables simultaneously. A deeper conceptual understanding and task types that focus on modeling and applying linear equation systems in the real world will help students overcome epistemological and didactic learning obstacles. This can also improve students' ability to understand and apply algebraic concepts in broader contexts (Hidayati et al., 2020).

### Linkage and Continuity of Tasks

The relationship between tasks and task continuity in mathematics textbooks is important to ensure that students acquire the necessary knowledge of algebraic concepts, especially in the material of linear equation systems of two and three variables (Gardenia et al., 2021). In this analysis, task continuity refers to how tasks are interconnected in constructing students' knowledge, from basic concepts to the application of more complex techniques. This is examined based on the REM, as illustrated in **Figure 2**, which outlines the expected progression of tasks and justifications that should be present in learning materials. A good arrangement of materials allows students to learn procedural steps more easily. Meanwhile, continuity between tasks can construct a strong conceptual understanding as the complexity of the material increases (Roy et al., 2020).

The presentation of the material in the early sections of the selected mathematics textbooks shows good continuity between tasks in presenting systems of linear equations in two variables. Simple activities such as $T_1$ and $T_2$ focus on a basic understanding of the relationship between two variables in a system of linear equations in two variables. At this stage, students are guided in learning how to find solutions to systems of linear equations in two variables. The presentation of tasks in $T_1$ to $T_4$ helps students build a solid understanding before being faced with more complex solution methods. The connectivity between tasks $T_1$ to $T_4$ helps support the formation of knowledge of algebraic material from basic to higher levels. The presentation of these tasks gives students time to adapt to more complex material (da Fonseca & Henriques, 2020).

The modeling of real-life problems, introduced in $T_5$, plays an important role in providing students with concrete experiences to connect algebraic concepts to real-world applications. Although there is only one task in $T_5$, this modeling task is a valuable enrichment for students as it allows them to understand how a system of linear equations in three variables works in real-world contexts. This modeling, though limited, provides an example of a real case where three variables can be used to represent three different entities in a problem, such as the prices of three different products or the analysis of three interrelated factors in a study.

Subsequently, $T_6$ offers more tasks (12 questions) that focus on solving systems of linear equations in three variables using the elimination or substitution methods. While $T_5$ presents only one modeling problem, $T_6$ delves deeper into the mathematical solution of systems of linear equations in three variables. In this case, $T_6$ serves as a logical continuation of $T_5$, where students learn to apply more complex elimination and substitution methods after modeling real-life problems. The instructions in $T_6$ tend to focus on applying procedural steps to eliminate one variable from the three equations so that students can find a solution for the system of three-variable linear equations.

However, despite $T_5$ and $T_6$ serving as enrichment parts, there are some weaknesses that need to be noted in this textbook. First, the number of tasks in $T_5$ is very limited, which does not provide enough opportunities for students to practice more in modeling real-world problems involving three variables. This makes the transition from two-variable to three-variable systems of linear equations feel less thorough. Students may become proficient at solving three-variable systems of linear equations after practicing in $T_6$, but their conceptual understanding of how real-world problems can be modeled into a system of three-variable linear equations may still be limited. The continuity between tasks in this regard becomes less optimal due to the significant difference between modeling tasks and solution tasks (Umiralkhanov et al., 2024).

While $T_6$ provides sufficient solution exercises, the lack of real-world context variety in $T_5$ may cause students to miss out on opportunities to explore three-variable systems of linear equations in various life situations. The textbook should include more modeling tasks in $T_5$, allowing students to gain broader experience in solving real-world problems with three-variable linear systems. The linkage between $T_5$ and $T_6$ needs to be sharpened by expanding the focus from merely procedural solutions to applying three-variable linear systems in broader contexts. Both $T_5$ and $T_6$ need to be enriched, especially in terms of modeling variety and context development, to create a more balanced learning experience between solution techniques and conceptual understanding in the real-world application of three-variable linear systems.

### Teacher Instructions and Potential Learning Obstacles

The instructions in the textbook predominantly adopt a mechanistic approach to teaching algebra, especially regarding solving systems of linear equations in two and three variables. Teachers are expected to adhere to highly structured procedural steps, such as the elimination and substitution methods, with minimal emphasis on the underlying concepts of these techniques. For example, although textbooks instruct students to eliminate one variable by adding or subtracting equations, they rarely explain the rationale for using this method. This has the effect that elimination techniques are often described as procedures to be followed without considering the relationship between arithmetic operations and the resulting solutions. It also leads to ignoring the need for a solid grounding in algebraic concepts.

This procedural emphasis is likely to create didactic learning obstacles. Students may view algebra only in terms of mechanical procedures without having a clear understanding of the deeper mathematical principles (Chevallard & Bosch, 2020b). Elimination techniques in selected textbooks are presented as a series of mechanical processes practiced in a very formalistic manner and often without regard to why or how the elimination of a variable shows direct relevance to the additive and distributive properties. Therefore, students find it difficult to understand why elimination and substitution methods work to find solutions. This tends to cause students to memorize rather than learn. Such misunderstandings can prevent students from completing algebraic tasks that require conceptual understanding, creativity, and non-routine problem solving (Brousseau & Warfield, 2020).

Epistemological learning obstacles can arise because there is an application of solution techniques without connecting algebraic concepts to real-life applications. Many textbooks, including those analyzed in this study, do not provide problems that illustrate how linear equation systems can be applied in everyday contexts (Pramesti & Retnawati, 2019; Utami et al., 2022; Wojongan & Jupri, 2023). Although elimination and substitution techniques are introduced operationally, there is little discussion of how these techniques can model real-world problems. Ensuring that the concepts taught in algebraic theory relate well to real-life situations is important to enable them to develop an appreciation of algebra in everyday life. Making connections between algebraic theory and its practical context is essential to helping students understand the relevance of algebra in everyday life. Without a clear context, students will perceive algebra as a very abstract discipline, disconnected from real-life experiences, and ultimately reduce their motivation to learn algebra (Gerami et al., 2024).

Excessive focus on procedures can also discourage students from trying other approaches to solving algebra problems (Utami & Prabawanto, 2023). Brousseau's (2006) theory of didactic situations (TDS) is based on the idea that if situations are not designed, learning will not occur (Brousseau & Warfield, 2020). TDS theory emphasizes that students should be in situations that encourage critical thinking and collaborative problem solving. However, the approach used in selected mathematics textbooks oversimplifies the problem-solving process and this eliminates the opportunity for students to learn different methods of solving systems of linear equations. If teachers rely too heavily on the procedural instructions found in textbooks, they may inadvertently discourage students from seeking alternative solutions or developing a deeper understanding of why certain techniques are used (Daher et al., 2022).

Textbooks that focus too much on procedural activities can hinder the development of a broader understanding of algebraic concepts. Algebra is not just about solving equations; it also involves understanding the relationships between symbols and their application to solving a variety of problems (Kieran, 2020). Unfortunately, students' understanding of algebra is often still limited to the use of procedural techniques, without exploration of how algebraic concepts can be integrated into the problem-solving process. Although students know how to use substitution or elimination, they may not yet understand how linear algebra can solve more complex problems in the context of geometry or economics. This lack of in-depth exploration can lead to a shallow understanding of algebraic concepts (Ayala-Altamirano & Molina, 2021).

Although mathematics textbooks provide comprehensive procedural guides for solving algebraic problems, they do not necessarily foster deeper conceptual understanding (Kuncoro et al., 2024). Teachers should focus more on connecting algebraic theory to real-world contexts and encouraging students to think critically about the concepts they learn (Scheiner et al., 2023). By doing this, students will be better prepared to tackle more complex problems and understand algebra not just as a set of procedural rules, but also as a powerful tool for solving real-world problems.

## CONCLUSION

The results of this study show how praxeological analysis can offer a more objective and in-depth way in analyzing mathematics textbooks, especially on the material of two linear equation systems. The results of the praxeological analysis show that the selected textbooks present a fairly good arrangement of tasks and offer problem-solving techniques. However, the selected textbooks have not sufficiently conveyed a deep conceptual understanding to students. Based on the praxeological analysis, it is known that the presentation of tasks on concepts that lead to three-variable linear equation systems requires careful evaluation. A more complete presentation of concepts is needed so that students do not rely too much on procedural methods and do not understand the reasons and theories behind the use of these methods, especially in complex algebra subjects. The potential for learning obstacles that arise in the presentation of selected textbook material can lead to epistemological learning obstacles and didactic learning obstacles.

We acknowledge that the methodology employed in this research only covers certain aspects of the textbooks, such as praxiological analysis related to tasks, techniques, technology, and theory in systems of linear equations in two and three variables. Other elements, such as the context of textbook use in the classroom and the presentation style of the material, have not been addressed in this study. Additional research is required, especially to broaden this reference model by applying it to textbooks from different contexts, including those published by private publishers and international sources. This aims to enrich the model with broader empirical data and identify a wider variety of didactic approaches.

The recommendation from the findings of this research is the need for the development of more balanced textbooks, which include additional tasks that facilitate a deeper conceptual understanding and emphasize the theoretical explanations behind the problem-solving techniques. Thus, textbooks should not only serve as tools for teaching procedural skills but also help students develop critical and analytical thinking abilities. The integration of procedural and conceptual aspects within textbooks will create a richer and more beneficial learning experience for students, preparing them to face more complex mathematical challenges in the future.

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