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Mathematics teachers' professional development: A systematic review of training methods

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Abstract

We present a systematic review of the Web of Science and Scopus in order to identify the training methods that have been used in promoting the professional development of early childhood and primary school teachers to teach mathematics. Using the preferred reporting items for systematic reviews and meta-analyses method, 58 papers have been identified. Six training methods have been identified: reflection on teaching practice (41.4%), creating learning environments (27.6%), content instruction (15.5%), lesson study (10.4%) and, finally, inquiry-based learning (3.4%) and flipped classroom (1.7%). We also confirmed that different training methods focus on different variables and factors such as knowledge, attitudes, beliefs, motivation, creativity, teaching vision, noticing or others.

Keywords: mathematics teachers, professional development, training methods, systematic review

INTRODUCTION

One of today's main concerns in mathematics education and in research in this field is to ensure that citizens are mathematically literate so they can better cope in a complex world in which mathematics is essential. Additionally, along with this applied role of mathematics, we must not forget its important formative and instrumental aspect (Niss, 2002; Organization for Economic Co-operation and Development [OECD], 2018). Ensuring that students progressively develop knowledge, abilities, skills, and attitudes (KASAs) consistent with this multiple role of mathematics requires, at the same time, that mathematics teachers develop professionally throughout their career.

Within the research agenda on the professional development of mathematics teachers, a large number of studies have investigated what variables and factors are involved (Llinares, 2018). Within this set of variables and factors, this study focuses on training methods to promote mathematics teachers' professional development for three main reasons, according to Even and Ball (2009a):

> A primary reason rests with teachers' central role in students' learning of mathematics, nonetheless, too often overlooked or taken for granted. Concerns about students' learning compel attention to teachers, and to what the work of teaching demands, and what teachers know and can do. A second reason is that no effort to improve students' opportunities to learn mathematics can succeed without parallel attention to their teachers' opportunities for learning. The professional formation of teachers is a crucial element in the effort to build an effective system of mathematics education. Third, teacher education is a vast enterprise, and although research on mathematics teacher education is relatively new, it is also rapidly expanding (p. 2).

In this paper, we take into consideration these reasons to investigate the following research question: what training methods have been used in promoting the mathematics teachers' professional development? This question leads us to our objective, which is to conduct a systematic review of the literature in the Web of Science (WoS) and Scopus databases from 1987 to 2023 to examine, select and determine the optimal bibliography to answer the research question formulated, following a systematic and well-defined process (Moher et al., 2015). The aim is to provide a framework of training methods to design intervention programs that promote the mathematics teachers' professional development.

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Contribution to the literature

- Studies investigating training methods to improve mathematics teachers' professional development have been increasing since 2016.
- Reflection, learning environments, content instruction, lesson study, inquiry and flipped classroom are the main training methods.
- Mathematics education feedbacks with other referential disciplines (psychology, pedagogy, sociology…) to improve mathematics teachers' professional development.

Mathematics teachers' professional development has been considered as a very important domain by 229 mathematics education researchers from 44 countries, making it one of the topics on which mathematics education research should focus over the next decade based on their opinions (Bakker et al., 2021). There are several pieces of evidence of the relevance given to this topic in recent years, such as the publication of monographs in mathematics education research journals indexed in journal citation reports:

> In Journal of Mathematics Teacher Education, for example, we have Video as a catalyst for mathematics teacher' professional growth, 2017, 20(5) and mathematics teachers as partners in task design, 2016, 19(2-3). In ZDM, we have Impact of university teacher education programs on teacher change and mathematics teaching practice, 2017, 49(2) and theoretical frameworks in research on and with mathematics teachers, 2013, 45(4). This series of monographs points to foci and perspectives on the mathematics teacher that focus the attention of researchers and teacher educators on questions centered on and about the mathematics teacher, his or her practice and on training processes (Llinares, 2018, p. 1).

Books have also been published by publishers that occupy top positions in rankings such as the scholarly publishers indicators in humanities and social sciences. At Springer, for example, we have *making sense of mathematics teacher education* (Lin et al., 2001), *the professional education and development of teachers of mathematics* (Even & Ball, 2009b), or *professional development of mathematics teachers* (Kaur et al., 2017). In Routledge, for example, we have *teachers' professional development and the elementary mathematics classroom* (Cohen, 2004) and *professional development and knowledge of mathematics teachers* (Zehetmeier et al., 2021).

This volume of contributions is advancing different research agendas on the learning and professional development of mathematics teachers, such as the variables and factors that influence the knowledge and skills useful for teaching mathematics or the relationship between theory and practice as an element for the professional development of trainers and researchers.

On the other hand, from the perspective of the training of mathematics teachers and their professional development, Ampaipipatkul (2004) notes that a training method includes the "activities that a trainer employ[ed] to convey knowledge, experience or information to the participants in order to facilitate their learning which might lead them to change their working behavior and attitudes" (p. 4). Deriving from this conceptualization, we adopt the definition of Martin et al. (2014):

> training method as a set of systematic procedures, activities, or techniques that are designed to impart KASAs to the participants that have direct utility in enhancing their job performance. It should be noted that in our definition, we do not require the inclusion of a trainer since some training methods can utilize instruction through sources other than a person (p. 12).

According to this, a relevant aspect to consider is the disposition towards this topic. Already in the 1990s, Elliott (1993) distinguished between technological practical professional development–which is interested in solving immediate problems–and reflective practical development, which deals with analyzing and substantiating before making decisions. This largely determines the types of training methods. Most of the contributions made from mathematics education involve this second approach, considering active training methods such as classroom research, design or analysis of class sessions or reflection on practice.

The inquiry based mathematics education (IBME), widely developed by the researchers (e.g., Artigue, 2017; Artigue & Blomhoej, 2013; Artigue et al., 2011), is mainly inspired by the foundations of problem solving (Pólya, 1945), realistic mathematics education (Freudenthal, 1973), the theory of didactic situations (Brousseau, 1997), or anthropological approaches, such as the anthropological theory of the didactic (Bosch & Gascón, 2009; Chevalard, 2007). Broadly speaking, Artigue (2017) states that mathematical inquiry starts from a question or a problem, and the answers are sought through observation and exploration; mental, material or virtual experiments are carried out; connections are sought with questions already answered and that have relevant analogies to the question being investigated; known mathematical techniques are put into play and adapted when necessary. This inquiry process is driven by, or leads to, hypothetical answers, to conjectures that

require validation. This is not usually a linear process. Often, initial conjectures are only true under specific conditions, which can lead to their revision, or even to questioning the definition of the mathematical objects involved. In addition, the process can lead to new questions and problems whose solution can affect the answers to the initial question, or even the formulation of the question itself. According to Artigue et al. (2011), the IBME

> [...] will improve students' mathematical understanding, which will result in their mathematical knowledge becoming more robust and functional in a diversity of contexts beyond that of the usual school tasks. It will help students develop mathematical and scientific curiosity and creativity as well as their potential for critical reflection, reasoning and analysis, and their autonomy as learners. It will also help them develop a more accurate vision of mathematics as a human enterprise, consider mathematics as a fundamental component of our cultural heritage, and appreciate the crucial role it plays in the development of our societies (p. 8).

The design or analysis of classroom sessions is another training method that has been used in mathematics education. It includes a variety of training methods that share the informed planning and implementation of mathematical tasks as a foundation for professional development: for example, task design, group dynamics techniques, lesson study (LS), flipped classroom, etc.

Various strategies have been used in the design of class sessions. Margolinas (2013), for example, refer to the design of mathematical tasks, which "affords opportunity to encounter mathematical concepts, ideas, strategies, and also to use and develop mathematical thinking and modes of enquiry" (p. 12); Alsina et al. (2019) have used group dynamics such as roleplaying, which consists of two or more people portraying a specific situation or case, acting out their assigned role such that said situation becomes more vivid and authentic; Bergmann and Sams (2012) introduced the flipped classroom, in which direct instruction moves from the collective learning space to the individual learning space, and the resulting space is transformed into a dynamic and interactive learning environment in which the trainer guides students as they apply the concepts, and can creatively engage in the subject matter.

In the analysis of class sessions, the LS is most notable. This approach began in Japan, and its name reflects the English translation of the original *jugyokenkyu*, a word composed of *jugyo* (lesson) and *kenkyu* (study or research). According to Fernández and Yoshida (2004), this perspective includes a particular approach to teaching practice whose ultimate goal is to improve both the teaching process of mathematics teachers and, as a result, student learning. According to these authors, this is achieved through two distinct phases, one involving observation and the other application, synonymous, respectively, with a theory taken from direct practice and subsequent experimentation with this theory. In this way, LS starts from practice-based knowledge that, once analyzed and compared, becomes a new form of knowledge. This resulting theoretical knowledge is again reconverted into practical knowledge, when applied in class as part of this methodology that, many times, becomes cyclical in its ability to adapt and constantly improve with respect to most of the problems that may come up in the classroom (Lewis & Tsuchida, 1999).

Reflection on practice is another training method that has had a broad impact on mathematics education. Ever since Schön (1983) put forth the idea of a reflective professional, other authors, primarily in the field of pedagogy (e.g., Korthagen, 2001; Melief et al. 2010; Perrenoud, 2001), have offered new strategies focused on reflection that have provided the basis for designing various professional development models grounded in these principles (Esteve & Alsina, 2024). The approaches of these models, which involve the perspective of learning based on the connection between experiences in practice and theoretical knowledge, start from the basis that teachers should learn multiple ways of acting, and then practicing them in real classroom contexts. They should have criteria to know when, what and why something is appropriate, and should reflect on it systematically (Korthagen, 2001). Therefore, reflective learning posits that an approach built on reflecting on practice, directly linked to the individual, and that emphasizes the connection between this practice and knowledge, is the essence of teacher training.

As mentioned, several authors have adopted these principles – which, as noted, originate mainly in pedagogy and also in sociocultural psychology (Vygotsky, 1978, 1986) – to promote the professional development of mathematics teachers and, more specifically, the transformation of teaching practice (Alsina, 2019). One of the main principles of these transformative models is that professional knowledge should be knowledge created by the trainee, from their own agency or identity, and not knowledge created beforehand by the trainer and conveyed by them. In other words, the person being trained does so by giving meaning to certain knowledge, and not by receiving that knowledge already imbued with meaning (Alsina, 2019). Based on this framework, the training processes identify the elements that allow pre-service teachers (PSTs) to assume goals, project their performance, observe it critically and evaluate their own strategies so as to formulate new improvement actions.

METHODOLOGY

In keeping with our objective, we conducted a review of the literature following the criteria and procedures of the quality standards of the preferred reporting items for systematic reviews and meta-analyses (PRISMA) statement, proposed by Moher et al. (2015).

The review has been organized into four phases:

1. establish search elements and Boolean logic,
2. select sources of information,
3. establish eligibility criteria, and
4. extract and manage the data to establish the sample.

Phase 1. Establish Search Elements and Boolean Logic

They have been formulated based on the key terms that guide the study, considering three central elements:

1. area of knowledge,
2. teacher training, and
3. search context: (mathematics) AND ("teacher education" OR "teacher training" OR "teacher professional development" OR "teacher learning" OR "teacher development" OR "teacher professional learning" OR "education preparation" OR "instructional training" OR "pedagogical development") AND ("improving teaching" OR "improving instruction" OR "enhancing teaching" OR didactic* OR "teaching method*" OR "teaching practices" OR "teaching techniques").

The search elements establish the use of the Boolean OR operator to include different formulations of the same concept, and the Boolean AND operator to combine the three elements that guide the research, focusing the search on studies that explicitly address the relationship between these elements.

Finally, the use of an asterisk (*) behind some terms indicates the truncation technique, used to find all the variant forms of a root word.

Phase 2. Select Sources of Information

The databases consulted include the most relevant scientific production at the international level in the field of educational research. The Clarivate analytics WoS and Elsevier Scopus databases were selected, given their related impact indexes (JCR and SJR, respectively), as well as their involvement in indexing scientific papers in journals that are housed under these parameters. Selecting the WoS and Scopus databases to carry out the systematic review guarantees an exhaustive, structured and reliable search of scientific literature, which is fundamental to obtain results in mathematics education that are valid and representative.

Phase 3. Establish Eligibility Criteria

The criteria are set out in **Table 1**. Scientific papers subject to a rigorous peer review process were considered, with book chapters, conference proceedings or other types of publications being excluded. The publication period considered was from 1987 to December 2023, given the advancement in research regarding the knowledge of mathematics teachers that took place starting with the contributions of Shulman (1987). We included:

(1) publications written in English, as it is the predominant language in educational research, and publications in Spanish, to take advantage of the linguistic competence of the authors and maximize the geographical coverage of the study,

(2) papers focused on early childhood and primary education, since our focus is on teachers who work at these school levels (6-12 years of age), or

(3) publications available for review, meaning publications with full text, excluding documents that we could not obtain for review.

Phase 4. Data Extraction and Processing

The Boolean logic described was applied to the titles, abstract and keywords of the documents, considering the eligibility criteria, which were filtered by the search engines of each database. The data were then exported to an MS Excel® spreadsheet. Subsequently, we read the titles and abstracts and reviewed the full texts. Using this process, the papers that did not meet the eligibility criteria in **Table 1** were excluded, along with duplicate documents based on a comparison of the paper titles and their digital object identifier number.

Sample

The sample was created based on the search process and resulted in 58 scientific papers that constitute the analysis units of our research (**Figure 1**, **Appendix**).

Table 1. Eligibility criteria

Criteria	Inclusion	Exclusion
Type of document	Peer-reviewed paper	Other formats
Publication period	1987 to 2023	Prior to 1987
Language	English and Spanish	Other languages
Level	Early childhood and primary education	Other levels
Access	Complete texts	Texts not available
Type of study	Intervention study	No intervention study

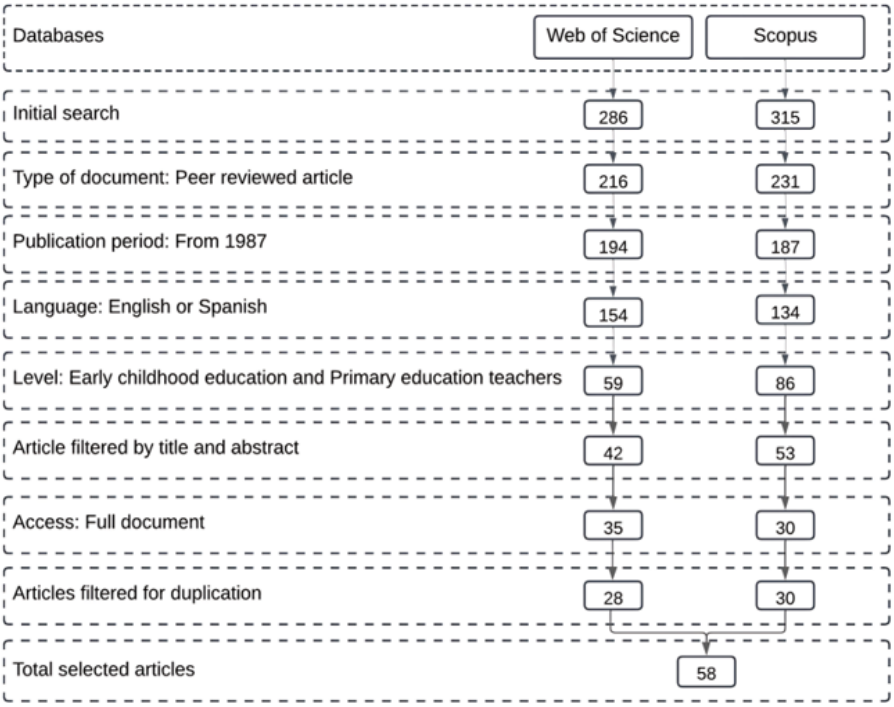

Figure 1. Flowchart of the process for selecting academic papers (Source: Authors' own elaboration)

Analysis Categories

To analyze the articles, we defined categories to approach and evaluate scientific productions. The analysis categories correspond to

(1) author(s), year and country where the study was conducted,

(2) participants: PST or in-service teacher (IST),

(3) Stage: early childhood education (E) or primary education (P),

(4) data collection method and instrument, and

(5) training methods.

Data Analysis

As noted, the data were analyzed by reading each paper and applying the established analysis categories by means of the content analysis technique (Krippendorff, 2013). To obtain a general categorization of each study, we conducted a vertical, or in-case, analysis (Miles et al., 2020) of each of the 58 papers. Next, multiple comparisons were made using a cross-sectional analysis in search of similarities and differences (Miles et al., 2020). Finally, we conducted a descriptive analysis of the data by preparing tables to analyze the qualitative information extracted from our review of the papers. We first present the general characteristics of the studies selected, and then the data on the different training methods used in the studies.

RESULTS

General Characteristics of the Papers

We explored the general characteristics of the 58 papers selected: year of publication; geographical distribution; participants and their training; and research methods and data collection instruments. The characteristics analyzed are presented below.

Year of publication

Figure 2 shows when the papers selected were published. Note that although the time interval between 1987 and 2023 was selected for our study, we only found published papers involving training methods for promoting mathematics teachers' professional development starting in 2006.

In general, there is a growing trend in the number of studies focused on this topic, starting in 2016 and peaking in 2021. As a result, the distribution over time

Figure 2. Number of articles published per year (Source: Authors' own elaboration)

shows that in the last eight years, the research community has shown great interest in improving the knowledge and attitudes of early childhood and primary school teachers to teach mathematics through the implementation of various training methods.

Geographical distribution

We show the geographical distribution of the studies, which were conducted in a total of 26 countries. Much of the research was concentrated in the United States (USA) (21.4%), followed by Spain (13.1%) and Turkey (10%). A smaller number of studies were carried out in countries such as China and Canada, with a presence of 6.7% and 5%, respectively, followed by Germany, Chile, Japan, Italy, Panama, and South Africa, with a presence of 3.3% in each case.

Finally, in the countries of Australia, Argentina, Brazil, Colombia, South Korea, Denmark, Finland, Hungary, Israel, Malaysia, Mexico, the Netherlands, the Czech Republic, Sweden, and Taiwan, we found evidence of a single study per country, resulting in a presence of 1.6%.

Participants and their training

Most of the studies (89.7%) focused on primary education teachers (e.g., Akkuş & Karakaya, 2020; Doruk, 2014; Giberti, 2022; Suh et al., 2021), while a smaller number (6.9%) involved early childhood education teachers (e.g., Alsina et al., 2023; Jenssen et al., 2022; Karatas et al., 2016; Laguna & Block Sevilla, 2022). It should be noted that only 3.4% of the studies involved both types of participants (e.g., Myers et al., 2020; Namukasa & Aryee, 2021).

Likewise, most of the studies (48.3%) involved ISTs (e.g., Broitman et al., 2023; Hoth et al., 2022), with 43.1% involving PSTs (e.g., Msimango et al., 2020; Ünlü; 2018), while 8.6% involved both PSTs and ISTs (e.g., Livy et al., 2023; Yang et al., 2021).

Research methods and data collection instruments

In general, 34 papers (58.6%) employed a qualitative research method, while 14 papers (24.1%) resorted to mixed research. Thus, research that opts to analyze and draw conclusions from unstructured and heterogeneous non-quantifiable data (qualitative) is more prevalent than research that interprets statistical data on a single variable (quantitative).

In relation to the techniques used in the different investigations, **Table 2** shows the predominance of interviews (50%) to analyze the training methods of early childhood and primary education teachers, followed by class observation and video recordings (32.8%) and questionnaires (27.6%). By contrast, written productions (13.8%), audio recordings, field notes, tests (10.3% each) and surveys (8.6%) are the least used instruments in the research analyzed.

Table 2. Techniques used in the various selected research works

Data collection techniques	*f* (%)
Interviews	29 (50.0)
Class observation	19 (32.8)
Video recording	19 (32.8)
Questionnaires	16 (27.6)
Written productions	8 (13.8)
Audio recording	6 (10.3)
Field notes	6 (10.3)
Test	6 (10.3)
Survey	5 (8.6)

In relation to the techniques used, practically half of the researchers (39.7%) conducted their study using one technique (e.g., Alsina et al., 2023; Novikasari & Dede, 2021; Yurekli et al., 2020); a smaller number of studies (31%) used two techniques (e.g., García et al., 2020; Youngs et al., 2022); 24.2% carried out their research using three techniques (e.g., Francis & Jacobsen, 2013; Pascual et al., 2021; Sebald et al., 2021); while 5.1% (Myers et al., 2020; Namukasa & Aryee, 2021; Swars et al., 2018) used four or more.

Consequently, we observed a variety of techniques that can be used to approach the various training methods employed by the studies in question to improve the knowledge and beliefs of pre- and in-service early childhood and primary mathematics teachers.

Training Methods

Six training methods have been identified that are implemented by the studies in question. **Table 3** shows the frequency of each method, considering all the papers selected: reflection on teaching practice (43.1%); creation of learning environments (27.6%); content instruction (15.5%); LS (10.4%); and, finally, inquiry-based learning (3.4%) and flipped classroom (1.7% each). The data corresponding to each training method are shown in **Table 3**.

Inquiry-based learning

This training methods was identified in only two studies, as shown in **Table 4**. The study of García-García et al. (2019) applies a pre-post questionnaire to analyze the beliefs, attitudes and teaching practices of 300 ISTs involving research-based learning, providing evidence of an improvement in the perception of barriers and obstacles that limit the use of this methodology; however, teachers' practices do not reflect its

Table 3. Training methods used in the papers selected

Training methods	*f* (%)
Inquiry-based learning	2 (3.4)
Creating learning environment	16 (27.6)
Flipped classroom	1 (1.7)
Content instruction	9 (15.5)
LS	6 (10.4)
Reflection of teaching practice	24 (41.4)

Table 4. Studies using the inquiry-based learning method

Author(s)	Country	Participants	Stage		Method/instrument	Main result
			E	P		
García-García et al. (2019)	Spain	300 IST		x	Mixed/questionnaire	Positive assessment of the professional development of teachers, despite a moderate transformation in their practices.
Laursen et al. (2016)	USA	13 PST		x	Mixed/survey, class observation, & interview	IBL can be used to analyze the MKT of pre-service teachers and expand their notions of how mathematics can be taught

widespread use. While Laursen et al. (2016) note a growth in the mathematical knowledge of 13 pre-service primary school teachers in a course designed based on the inquiry-based learning methodology to reflect on teaching practice.

Creating a learning environment

Sixteen studies were identified that focus on developing instructional practices for mathematics class with help from different data collection instruments, including questionnaires, audio recordings, interviews, and others (**Table 5**).

For example, the study by Copur-Gencturk et al. (2019), in the context of a Professional Development course, delves into five categories of teaching practices:

(1) observing and being observed,

(2) planning classroom implementation,

(3) examining student work,

(4) presenting, leading, and writing, and

(5) solving mathematics problems.

The results reveal that focusing on knowledge of curriculum content and examining student work was highly correlated to teacher learning. Along the same

Table 5. Studies using the creating learning environments method

Author(s)	Country	Participants	Stage		Method/instrument	Main result
			E	P		
Alsina et al. (2023)	Spain	105 PST	x		Mixed/written productions	Transformation of teachers' mathematical knowledge in the planning and implementation of teaching activities
Castellanos et al. (2017)	Colombia	12 PST		x	Qualitative/audio and written productions	Optimization of the reflection process in the teacher training process
Castro et al. (2022)	Chile	18 PST		x	Qualitative/ questionnaire	Change in the perception and understanding of the design of activities that promote problem-solving
Copur-Gencturk et al. (2019)	USA	203 IST		x	Quantitative/test	Improved mathematical knowledge of teachers
Francis and Jacobsen (2013)	Canada	10 IST		x	Qualitative/video, audio and interview	Opportunities for collaborative professional development online become more viable
García et al. (2020)	Spain	108 PST		x	Qualitative/interview and questionnaire	Increased motivation, improved creativity, and development of mathematical proficiency
Goldrine et al. (2015)	Chile	39 PST		x	Mixed/video, class observation, interview, written productions	Improvement in the mathematical knowledge for teaching numeracy in early childhood education
Gyöngyösi-Wiersum et al. (2019)	Hungary	17 PST		x	Qualitative/questionnaire	Improved understanding of mathematics
Jao et al. (2018)	Canada	44 IST		x	Qualitative/video	Refinement of teaching practice
Johnson et al. (2022)	USA	1 IST		x	Quantitative/class observation and interview	Transformation of the teaching practice of mathematics and analysis of the concept of measuring angles
Livy et al. (2023)	Australia	176 PST/ IST		x	Quantitative/survey	Reflection on the opportunities provided by the incorporation of children's literature to enhance the teaching of mathematics
Marimón et al. (2021)	Panama	93 PST/IST		x	Mixed/video	Improvement in didactic analysis skill to assess the didactic sequences developed
Myers et al. (2020)	USA	13 IST	x	x	Mixed/class observation, interview, written productions	The teachers manage to establish connections between learning during the training process and their classroom instruction practices
Oonk et al. (2020)	Netherlands	269 PST		x	Qualitative/video and interview	The use of theory and its link to teaching practice is influenced by the teachers' previous mathematical training
Rodrigues et al. (2022)	Brazil	15 PST		x	Qualitative/video, class observation and interview	Comprehensive and evolutionary development of the process of reflecting on teaching practice in the mathematics classroom
Ünlü (2018)	Turkey	41 PST		x	Mixed/video, field notes and interview	Positive effects on the teachers' belief in self-reliance and on their mathematics teaching skills

lines, through the development of microteaching practices, Ünlü (2018) identifies positive effects on the teaching skills of PSTs. Johnson et al. (2021) analyze the case of a primary school teacher who participates in a professional development program for one year to aid in the implementation of instruction and encourage the mathematical reasoning of students. Based on pre-post interviews and different in-classroom observations during lessons on measuring angles, the teacher developed her practice in four key areas: vision or foundation, evidence, students and content. The results show both the transformation of her teaching vision and the interpretation of students' mathematical learning. In this regard, the authors reflect on the importance of pedagogical empathy, which provides a tool to encourage and guide students by generating positive opinions about their abilities; in other words, the different forms of student work can become something valued and nurtured.

Flipped classroom

This training method was identified only in the study by San Cristobal et al. (2017), as shown in **Table 6**. The study by San Cristobal et al. (2017) implemented the flipped classroom methodology in an experimental group with 19 pre-service early childhood teachers to analyze aspects of early childhood education, while a control group with 43 PSTs followed an expository teaching model. The results show that this methodological strategy increases the performance of the PSTs; moreover, the subjective evaluations obtained in a questionnaire revealed that they were motivated and satisfied by the methodology in question.

Content instruction

We identified nine studies that, by administering questionnaires or tests, delve into general methods to promote the instruction of a wide range of mathematical content (**Table 7**).

For example, Zhou et al. (2006) inquire about the teaching of fractions with in-service American and Chinese primary school teachers, evaluating the application of their mathematical knowledge based on the contributions of Shulman (1987). The results show that American teachers are significantly behind Chinese teachers in identifying important points involving both teaching concepts of fractions, calculations, and posing problems, as well as ensuring student understanding. Passarella (2021) discusses instruction on modelling, showing that teachers regularly included some aspects of the modelling process in their classroom activities, in terms of using real contexts as initial situations for mathematics lessons and showing the real applications of mathematics. However, they request more materials

Table 6. Studies using the flipped classroom method

Author(s)	Country	Participants	Stage		Method/instrument	Main result
			E	P		
San Cristóbal et al. (2017)	Spain	64 PST		x	Quantitative/ questionnaire	This pedagogical approach boosts the performance of the PSTs, as well as their motivation and satisfaction with this methodology

Table 7. Studies using the content instruction method

Author(s)	Country	Participants	Stage		Method/instrument	Main result
			E	P		
Giberti (2022)	Italy	129 IST		x	Qualitative/questionnaire	Enrichment of the knowledge of the pedagogical and mathematical content of IST
Heyd-Metzuyanim et al. (2016)	Israel	16 PST		x	Qualitative/video and field notes	The instruction proposed by the pre-service teachers reflected an exploratory approach to the content and did not delve into mathematical ideas
Jansen et al. (2017)	USA	6 IST		x	Qualitative/class observation and interview	Incorporation of instructional practices by teachers to teach mathematics
Jenssen et al. (2022)	Germany	129 PST	x		Quantitative/test	The teachers' knowledge increases over time; however, their beliefs do not change.
Pascual et al. (2021)	Spain	1 IST		x	Qualitative/video, class observation & interview	Similarity between the difficulties of pre-service teachers when teaching geometry and those exhibited by primary school students
Passarella (2021)	Italy	107 IST		x	Mixed/questionnaire	The teachers implement some aspects of mathematical modeling in their classes, while problem solving is more integrated into classroom work
Yang et al. (2021)	Taiwan	287 IST/PST		x	Mixed/questionnaire	The lack of knowledge of pedagogical teachers prevents the integration of children's literature into teaching of math
Youngs et al. (2022)	USA	83 IST		x	Mixed/survey and class observation	The teachers' management of their classrooms is positively correlated with the implementation of ambitious instruction
Zhou et al. (2006).	USA and China	132 IST		x	Qualitative/interview and questionnaire	USA teachers are significantly behind their Chinese counterparts in identifying important points involving teaching concepts of fractions, calculations, and posing problems, as well as ensuring student understanding

to aid in their preparation and practice. Heyd-Metzuyanim et al. (2016) focus on instruction in algebra, revealing that the instruction proposed by PSTs reflected an exploratory approach to the content and did not delve into mathematical ideas.

Finally, Youngs et al. (2022) ensure that opportunities to learn general principles of instruction and general teaching methods during teacher preparation were positively associated with the implementation of ambitious mathematical practices by teachers. By contrast, they show that PSTs need opportunities in all content areas to learn how to plan lessons and units, teach and model skills and strategies, and provide feedback to students.

Lesson study

Our systematic review identified six studies (**Table 8**). The data were collected using a variety of strategies and instruments, such as video recordings, lesson observations, reflections, and interviews.

In general, the studies show a progression in the professional development of teachers (e.g., Lim et al., 2016; Pang, 2016; Suh et al., 2021). Rasmussen (2016), for example, states that the interaction between teachers shapes the development of their discourse on mathematical learning. Similarly, teachers notice both positive exemplifications and opportunities for growth in their implementation of using and connecting mathematical representations, specifying learning objectives for students, designing mathematical tasks in a rigorous and meaningful manner, and designing the lesson structure to maximize whole-class participation by posing purposeful questions and supporting more productive work by students (Pang, 2016; Suh et al., 2021). More specifically, Lim et al. (2016) identify five changes they observed in teaching practices:

(1) the innovative use of teaching materials,

(2) choice and sequencing of learning tasks,

(3) training students to create questions,

(4) changing group sizes, and

(5) greater teacher participation.

Reflection on practice

We found 25 papers that focus on the exchange of ideas and information between participants, with the aim of reflecting on their educational practices (**Table 9**). Most of the data are collected using a wide variety of strategies and instruments, such as class observations, interviews, video analysis, questionnaires, and others. For example, Pourdavood et al. (2021) analyze a summer course on the integration of mathematics teaching. In this course, the participants summarized, analyzed and presented their opinions on different readings and viewed six films on learning theories and constructivist teaching and learning in order to reflect on the teaching of mathematics.

Table 8. Studies using the LS method

Author(s)	Country	Participants	Stage		Method/instrument	Main result
			E	P		
Asami-Johansson et al. (2020)	Japan, Finland, Sweden	107 PST		x	Mixed/video and questionnaire	Japanese teachers focus on very specific mathematical and didactic objectives in their lessons, while Finnish and Swedish teachers have broader goals
Lim et al. (2016)	Malaysia	5 IST		x	Qualitative/video, class observation and interview	There are changes in teaching practice, such as the innovative use of teaching materials, choice and sequencing of tasks, greater teacher participation, among others.
Pang (2016)	South Korea	17 IST		x	Qualitative/video, interview and written productions	Teachers embrace the importance of detailed planning and develop a better understanding of how to use student ideas
Rasmussen (2016)	Denmark	8 PST		x	Qualitative/audio, field notes	Changes in teacher discourse on post-classroom reflection, development of specific knowledge associated with teaching practice
Shinno and Yanagimoto (2023)	Japan	8 PST/IST		x	Qualitative/video, class observation and interview	The teachers' work of preparing for lessons is conditioned by institutional or educational limitations
Suh et al. (2021)	USA	18 IST		x	Qualitative/video	Opportunities for teachers to grow in relation to the use of mathematical representations, questioning and supporting student productivity

Table 9. Studies using the reflection on practice method

Author(s)	Country	Participants	Stage		Method/instrument	Main result
			E	P		
Akkuş and Karakaya (2020)	Turkey	5 IST		x	Qualitative/classroom observations and interview	The teachers consider various characteristics in terms of implementing their pedagogical objectives
Broitman et al. (2023)	Argentina	10 IST		x	Qualitative/audio, class observation and interview	Reflection on strategies for teaching mathematics to students with disabilities.

Table 9 (Continued). Studies using the reflection on practice method

Author(s)	Country	Participants	Stage		Method/instrument	Main result
			E	P		
Burgos et al. (2020)	Spain	93 PST		x	Mixed/video	Development of the ability to analyze the didactic relevance of videos on proportionality
Coles (2019)	USA	7 IST		x	Qualitative/video and audio	Teachers begin to focus and discuss the details of the mathematics class, which they analyze in videos
Dofková (2019)	Czech Republic	77 PST		x	Quantitative/ questionnaire	PSTs evaluate their readiness for future mathematics instruction positively
Doruk (2014)	Turkey	13 PST		x	Qualitative/class observation and interview	PSTs generally adopt traditional teaching approaches
Hoth et al. (2022)	Germany	131 IST		x	Quantitative/survey and test	The ability to quickly identify typical student errors is highly dependent on the teachers' knowledge of the mathematical content
Huang et al. (2022)	China	2 IST		x	Qualitative/video and interview	For the experienced teacher, students' mistakes in homework and her online teaching practice triggered her knowledge changes. For the young teacher, the online video lessons, relevant resources online and student performance were the primary sources that triggered the changes in her knowledge for teaching
Karatas et al. (2016)	Turkey	139 IST	x		Quantitative/ questionnaire	The more experienced teachers have a better understating of the curriculum and the children
Kinser-Traut and Turner (2020)	USA	1 PST		x	Qualitative/field notes, interviews and written productions	Changes made in the knowledge and practices related to children's mathematical thinking and linguistic, cultural and family knowledge
Laguna and Block Sevilla (2022)	Mexico	2 IST	x		Qualitative/video, class observation and interview	Transformation and enrichment of a didactic proposal for an early childhood education classroom
Mabova et al. (2022)	South Africa	41 IST		x	Qualitative/class observation and interview	Improvements in teachers' classroom practice and critical reflection on their teaching methods
Marimón and Diez-Palomar (2023)	Panama	5 IST		x	Qualitative/interview	Critical reflection by teachers on educational practice, showing that socio-professional autonomy and practicality generate positive emotions in students
Mendías (2021)	Spain	488 PST		x	Quantitative/ questionnaire	Average levels of anxiety and self-confidence towards mathematics are apparent
Msimango et al. (2020)	South Africa	12 PST		x	Qualitative/interview	PSTs develop a knowledge of mathematical and pedagogical content through classroom practice that is influenced by mentors
Namukasa and Aryee (2021)	Canada	12 IST	x	x	Qualitative/class observation, field notes, interview and questionnaire	Thriving in Montessori mathematics classrooms requires teachers to develop pedagogical knowledge
Novikasari and Dede (2021)	Turkey	414 PST		x	Qualitative/ questionnaire	Teachers' subjective beliefs about multiplication objectively influence the knowledge they possess.
Pascual et al. (2023)	Spain	30 PST		x	Design inquiry/video and written productions	The mobilization of didactic knowledge of the content is more evident when analyzing a class
Pepin et al. (2017)	China	3 IST		x	Qualitative/class observation, field notes and interview	The resources teachers use (textual, digital, human) influence their teaching identity and practice
Pourdavood et al. (2021)	USA	47 PST/IST		x	Qualitative/written productions	The exchange of ideas gives teachers opportunities to acquire more knowledge and confidence in integrating mathematics
Sebald et al. (2021)	USA	18 IST		x	Qualitative/audio, class observation and interview	Collaborative practices improve teachers' mathematics skills and professional development
Swars et al. (2018)	USA	32 IST		x	Mixed/survey, class observation, interview and test	Greater opportunities are required to improve teachers' knowledge of specialized content
Yurekli et al. (2020)	Turkey	9 PST		x	Qualitative/interview	Improving teachers' self-efficacy is related to their experience and emotional state
Zhang et al. (2021)	China	9 IST		x	Mixed/interview and questionnaire	Greater understanding of the process of informal interactions between teachers to enhance professional development

The results revealed that the exchange of ideas and information, together with support from the instructor, provide faculty and PSTs with opportunities to acquire more knowledge and confidence in planning their classes and teaching. Along the same lines, Mabova et al. (2022) examine pedagogical practice before and after a pedagogical intervention program by analyzing and discussing the teaching methods of the participants. Based on an analysis of documents, discussion groups, interviews and class observation, the teachers were able to reflect more critically on their teaching methods. The participants made some improvements in practice, in class observations after the development program. In the study conducted by Pepin et al. (2017), two rounds of in-depth interviews, observations and representations of teachers were carried out in order to analyze their mathematics teaching skill and how to develop it using the resources available to each teacher in their practice. The results show the importance of sharing resources to improve the instructional practices of all members, as in the case of Jiang, who shared and acquired increasingly detailed knowledge about different resources. From this perspective, collaboration between teachers is essential to improve their teaching practices (Zhang et al., 2021).

DISCUSSION

In this paper, we have presented a systematic review of the literature in the WoS and Scopus databases from 1987 to 2023 in order to identify the training methods that have been used in promoting the mathematics teachers' professional development.

Delving into these different contributions requires an exhaustive analysis, since there are many subtleties that must be considered in order to avoid making superficial interpretations or erroneous categorizations. For this reason, we decided that a systematic review could offer a more objective analysis framework (Moher et al., 2015).

The first contribution of our systematic review is that it has made it possible to categorize the training methods that have been used in various studies to promote the mathematics teachers' professional development. Specifically, we identified six methods: reflection on teaching practice (41.4%); creation of learning environments (27.6%); content instruction (15.5%); LS (10.4%); and, finally, inquiry-based learning (3.4%) and flipped classroom (1.7%).

A relevant aspect to note is that, although the 58 papers in the sample are included in the research agenda on the professional development of mathematics teachers, the origin of the six training methods identified comes from both mathematics education itself and other disciplines, most notably pedagogy and psychology. The methods identified highlight the global dialectical view of mathematical education. This perspective emphasizes its role as both a scientific discipline and an interactive social system encompassing theory, development, and practice. These training methods further demonstrate the permeability and feedback loop between mathematical education and other relevant disciplines, all of which contribute to improving teaching practice (e.g., Higginson, 1980; Steiner, 1985). These relationships are discussed below:

Learning by inquiry, which focuses on research in the classroom and problem solving, has been widely developed since research on mathematics education by the researchers (e.g., Artigue, 2017; Artigue et al., 2011; Artigue & Blomhoej, 2013), as already described in the theoretical framework. However, as evidenced in the study by García-García et al. (2019), we must not forget the imprint of the sociocultural perspective, especially with regard to the idea of a community of practice (Lave & Wegner, 1991).

The creation of learning environments, as evidenced in the systematic review, is a strategy that is widely used to improve teacher knowledge to teach mathematics through various activities (stories, role plays, etc.). In sociocultural psychology, the concept of teaching-learning activity entails a set of training methods that involve people who interact guided by the same motive or purpose in a given context. Vygotsky (1986) alludes to this type of strategy when he considers that the teaching-learning process is collaborative, reciprocal and governed by a shared purpose. This vision of activity, in which the trainer and the people who are being trained actively collaborate, was observed in the 16 papers categorized within this strategy.

In the flipped classroom, first documented by two American chemistry teachers (Bergmann & Sams, 2012), direct instruction moves from the collective learning space to the individual learning space, and the resulting space is transformed into a dynamic and interactive learning environment in which the trainer guides students as they apply the concepts and can creatively engage in the subject matter. This methodology, which was applied to mathematical education in the study by San Cristóbal et al. (2017), also permeates various sociocultural contributions, especially the transition from the inter-psychological to the intra-psychological by the people who are being trained, or the mediation carried out by the trainer (Vygotsky, 1986).

Content instruction relies on the different models of teacher knowledge to teach mathematics (e.g., Ball et al., 2008; Rowland et al., 2005). In turn, these models were fueled by the contributions of Shulman (1987) who, from psychology, characterized the basic knowledge for teaching the disciplines: content knowledge, general pedagogical knowledge and pedagogical content knowledge. The nine studies identified in the review exemplify how, in one way or another, a task or sequence of tasks oriented around a teacher's knowledge model can contribute to the development of this knowledge in initial or continuous training.

LS, with six papers, is another of the training methods identified in the review that contributes to the professional development of mathematics teachers. This methodology, which, as noted in the theoretical framework, originated in Japan, has had a wide impact in the USA and recently in Europe thanks to pedagogues such as Lewis (the USA), Elliott (the United Kingdom), and others. Some essential features of this methodology that seeks the continuous improvement of educational practice are to define a problem, design, collect evidence, analyze, discuss and reflect collaboratively (e.g., Fernández & Yoshida, 2004), which constitutes evidence of the dialogue between various referential disciplines for mathematical education.

Finally, reflection on practice traces its roots back the furthest, all the way to Greek philosophy. In the modern era, authors such as the philosopher of education D. Schön, the sociologist F. Perrenoud, and the pedagogue F. Korthagen, among others, have endorsed reflective learning to promote the professional development and the transformation of teaching practice. Thus, philosophy, sociology and pedagogy, among other disciplines, have contributed to developing this perspective, which has been widely implemented in mathematics education. The 25 papers identified in the systematic review are evidence. These papers, which, as already noted in the theoretical framework, rely on the perspective of learning based on a connection between practical experience and theoretical knowledge, emphasize that teachers should learn multiple ways of acting and practice them in real classroom contexts; they should have criteria to know when, what and why something is appropriate and they should reflect on it systematically (Korthagen, 2001).

CONCLUSIONS

The data obtained through the systematic review provide, as a whole, a relevant contribution to the community of researchers in mathematics education. Along with detecting, obtaining, consulting, reviewing and examining the 58 papers that comprised the analysis unit by means of a systematic and well-defined process, in our case the one offered by the PRISMA method (Moher et al., 2015), the review carried out has made it possible to identify how the data were obtained and interpreted, as well as to describe the design of the studies analyzed, with the purpose of offering a critical evaluation of the papers published and obtaining conclusions based on the evidence presented in said papers, thus answering the research question posed.

As a whole, the main conclusions of the systematic review are, as follows:

1. Various descriptive aspects of the articles included in the review (year of publication, geographical distribution, participants and their training and research methods and data collection instruments) have been specified. In this regard, an interesting aspect is that the interest in characterizing professional development occurred in the 1980s (Shulman, 1987), while the first specific models for identifying the variables and factors involved in mathematics teachers' professional development emerged in the first decade of the 21st century (e.g., Ball et al., 2008; Rowland et al., 2005). However, it was not until 2016 that an increase was observed in the number of studies that go beyond promoting the mathematics teachers' professional development based on the various training methods identified.

2. Six training methods have been identified to improve mathematics teachers' professional development, showing that the reflection on teaching practice method is the most used training method; in contrast, the systematic review has shown the underuse of inquiry-based learning and flipped classrooms. These data have implications for mathematics teacher education, as they show that methods that most defend and emphasize the autonomy of PSTs in order to promote professional development have not yet been consolidated.

3. Some methodological limitations have been identified that may explain the dominance of certain methods: for example, the article on flipped classroom uses only a quantitative methodology, when in most studies a qualitative or mixed methodology is used; additionally, none of the articles on flipped classroom or inquiry-based learning use video and written productions as data sources, even though they are widely used in the other methods to promote mathematics teachers' professional development.

4. Finally, the six training methods to promote the mathematics teachers' professional development focus on different variables and factors such as knowledge, attitudes, beliefs, motivation, creativity, teaching vision, noticing or others.

Some of the main limitations of the study are the selection of search terms, along with the inclusion and selection criteria of the studies, which may have prevented locating other relevant articles. Further research will thus be necessary in future studies to specify critical aspects of this study, such as including new search terms that were not considered in this review, such as mathematics specialist or mathematics coach, in addition to also considering teaching studies (a newer form in situ PD).

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APPENDIX

Table A1. Articles used for analysis

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