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Mathematical and neuro-mathematical connections activated by a teacher and his student in the geometric problems-solving: A view of networking of theories

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Abstract

The research goal is twofold: to articulate neuro-mathematics with the extended theory of mathematical connections that uses onto-semiotic approach tools and to explore the connections established by a teacher and his student when solving a problem about the volume of two boxes, one of toothpaste and the other of tomato. This research was developed in two stages: the theories were articulated assuming concordances and complementarities, highlighting the notion of connection, and a context of reflection was considered carried out in three phases where the participants were selected, participant observation was carried out in the classroom during solving a problem and then analyzing the data with the new tool to explore mathematical and neuro-mathematical connections. The findings present the mathematical connections established by the teacher and the student of meaning, feature, procedural, different representations (alternate, equivalent, and from a horizontal mathematization view), and part-whole, as well as neuro-mathematical connections of: recognition of terms and symbols; visual perception, spatial skills and motor coordination; association of mathematical concepts and formulas; intermediate calculations and unit conversion; solve operations step by step and understand the process; verification and conclusion, activated in the brain areas linked to each mathematical practice sequentially.

Keywords: mathematical and neuro-mathematical connections, volume of solids, teacher, student, mathematics education

INTRODUCTION

In the mathematics education field, various curricular organizations are recognized that point out that establishing mathematical connections is important for students to understand the different mathematical concepts addressed by their teacher (Departament d'Ensenyament, 2017; Ministry of Education [MOE], 2006; Ministry of National Education [MEN], 2006; NCTM, 2000; Rodríguez-Nieto et al., 2024).

Just as connections are suggested in curricular approaches, they are also suggested in various investigations in which approaches to specific mathematical concepts are distinguished that, on many occasions, students and some teachers present difficulties in understanding which are caused by the lack of necessary connections that the student does not establish in the resolution of a problem (Businskas, 2008; Mhlolo, 2012; Rodríguez-Nieto et al., 2021a).

In this context, most research focuses on exploring the connections on algebra concepts (e.g., the work of Businskas, 2008) focused on the connections of mathematics teachers to the treatment of the quadratic equation); in calculus as the understanding of

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Contribution to the literature

- It contributes to the literature with the networking between the extended theory of mathematical connections (ETC) (articulated with the onto-semiotic approach [OSA]) and the cognitive tools of neuro-mathematics.
- The mathematical and neuro-mathematical connections of a student and his teacher are reported when they solve an extra-mathematical and geometric problem, where similarities were found in their mathematical practices.
- The analysis of mathematical connections is more detailed if it is executed with the conceptual and cognitive tools of neuro-mathematics, that is, they are done with the establishment of neuro-mathematical connections.

exponential and logarithmic functions through connections (Campo-Meneses & García-García, 2023), the derivative and the integral with pre-university students emphasizing the reversibility of these inverse functions (García-García & Dolores-Flores, 2018, 2021), analysis of the understanding of the derivative of university students from a networking of theories (ETC-OSA) (Rodríguez-Nieto et al., 2021a, 2021b, 2021c, 2022a), identification of the pre-university students' connections on the concepts of rate of change (Dolores-Flores et al., 2019) and the different conceptualizations of the slope.

Other research focuses on the connections made by pre-service teachers in solving geometric area problems of plane figures (Caviedes-Barrera et al., 2023), on how the connections influence the teacher's knowledge about the concept of function (Hatisaru, 2022). Particularly, in primary education it is considered essential for students to establish mathematical connections. For their part, Kenedi et al. (2019) stated that:

> The ability of students to connect mathematically is one of the essential things that must be achieved by students in the learning process because if students know the relationship between the concepts, they will quickly understand the mathematics itself and open opportunities for students to develop their mathematical skills (p. 70).

De Gamboa et al. (2023) conducted a study to understand how mathematical connections emerge in the classroom where interactions between the teacher and students are encouraged that are related to concepts, properties, procedures, proofs and suggest that studies on connections should provide evidence on extra-mathematical connections. Bortoli and Bisognin (2023) explored the mathematical connections established by undergraduate mathematics students in solving problems about arithmetic progressions, highlighting patterns, regularities, and representations.

Going deeper into this important topic, the connections have been explored from sociocultural approaches to identify ethnomathematical connections from a STEAM vision in different everyday practices (Rodríguez-Nieto & Alsina, 2022; Rosa & Orey, 2021). Likewise, ethnomathematical connections were articulated with ethnomodeling to explore gastronomic activities such as the making of meat tops and Mexican tacos (Rodríguez-Nieto et al., 2022). However, from the theory of mathematical connections worked in the field of mathematics education, a study concerned with what happens in the person's mind before establishing mathematical connections (operational and discursive) is not recognized, at least by Osler and Mason (2016) investigated the use of brain-based neuroscience in mathematical terms, considering, in turn, the mathematical law of trichotomy (Osler, 2012) and Giraldo-Rojas et al. (2021) proposed an articulated approach to neuro-mathematics in the construction of a geometric body and its detailed analysis from APOS theory. Along these lines, we choose to take a tour of the considerations about connections in neuroscience, neuroeducation, neuro-pedagogy and the neuro-mathematical approach such as the language of the brain.

It is timely to mention that there is a specialized literature on neuroscience and mathematics, for example, neuroscience has contributed significantly to the cognition and learning of mathematics (Hackett, 1985; Lent et al., 1993; Obersteiner et al., 2019; Olkun, 2022; Rivera-Rivera, 2019; Tuttle & Davidesco, 2023; Verschaffel et al., 2016). In these cognitive studies it has been shown that the brain works abstractly, and the numerical interpretation is worked from neuroimaging (Cohen & Walsh, 2009). According to advances in neuroscience, it has been considered that the left hemisphere is where there is greater recognition of both spoken and written language that is involved in numeration, mathematics, and logic (De la Serna, 2020).

The brain, through the senses, collects and analyzes external and internal information, and processes it and emits a response, after the attentional filter is overcome. The information received by the brain is distributed and analyzed separately to then be integrated and compared with existing memory traces and thereby generate new knowledge (De la Serna, 2020, p. 32). On the one hand, in the case of mathematical attention, the bilateral posterior-superior parietal system intervenes, which enables spatial and non-spatial orientation in the system of mental representation of quantities (De la Serna, 2020). On the other hand, in the parietal lobe is the ability to understand written language and solve mathematical problems. This area develops cognitive functions such as attention, numerical processing and working memory, therefore, when this lobe is affected, the appearance of dyscalculia will occur (problems with mathematics that account for semantic alterations about quantities, deficit in the understanding and expression of numbers and problems in mathematical calculations) (Cantillo-Rudas & Rodríguez-Nieto, 2024; Cohen & Walsh, 2009; Rousselle & Noel, 2007; Rubinsten & Henik, 2005).

In relation to the neural bases, the compression and expression of number verbally is an operation located in the language area, in the dominant hemisphere, normally the left, in the angular gyrus. For its part, the representation of numbers is processed in the middle ventral occipito-temporal cortex and in the fusiform gyrus. With respect to the abstract representation of quantities, the intraparietal sulci are involved in a bihemispheric manner. It is considered that, in the processes of numerical understanding and calculation, an entire distributed network is established at the neural level where there is a distribution of different tasks, which are accompanied by the analysis of the stimulation, the identification of the stimulus, the assignment of value and quantity, and its manipulation, and all of this, prior to the pronunciation of the word corresponding to said quantity (De la Serna, 2020).

Dehaene et al. (2003) examined the neuroanatomical systems responsible for processing different mathematical operations (addition and subtraction), and within their findings, they established that people represent numbers in a mental quantity line. Furthermore, mental arithmetic, particularly subtraction and division, activate intraparietal sulci, which are involved in quantitative processes related to the number line (Lee & Fong, 2011). In this sense, the parietal cortex plays a crucial role in low-level calculation, symbolic and non-symbolic representation, and automatic execution after extensive practice, while the prefrontal cortex is involved in more complex calculations and algorithmic processing (Chein, 2012; Cohen Kadosh et al., 2007, 2010; Hawes & Ansari, 2020).

In recent years, there has been great interest in research into the neural substrates of mathematical cognition and education, and frontal and parietal regions have been repeatedly highlighted as key regions (Arsalidou et al., 2018; Menon, 2016). At a neurobiological level, the lack of mathematics education could affect neuronal changes in regions that are involved in the acquisition of mathematics skills, mainly in frontoparietal regions (plasticity account). This process may be favored by concentrations of neurotransmitters that preceded the anatomical changes.

Regarding the most popular cognitive models of numerical processing, the triple code model is mentioned (Dehaene & Cohen, 1995; Dehaene et al., 2003), which exposes three different codes of mental representations at the level of mathematical cognition, and which depend on the task to be solved. According to the triple code model called "neuro-functional" (Dehaene & Cohen, 1995), there are three instances in which numbers are mentally manipulated. The first system of the proposed triple code model is related to quantity and is commonly known as "number sense" in cognitive theory, which employs a non-verbal semantic representation of size and distance associations between numbers on a number line. Additionally, it facilitates comparisons between magnitude (e.g., more vs. less) and approximation (e.g., estimation) and recruits the right and left sides of the intraparietal sulcus with a brain structure that has previously been associated with processing of numerical information among other cognitive functions.

The second system, the verbal system, represents numbers in a verbal format (e.g., lexically, phonologically, and syntactically), being a mental schema, which is activated when arithmetic facts become familiar and learned through rote learning, like addition and multiplication tables. It has also been proposed that the visual system is involved in the spatial representation and manipulation of numbers in symbolic format (e.g., Arabic numerals). It is recruited in tasks that require spatial attentional orientation, such as number comparison, approximation, subtraction, and counting. The third system is the superior parietal lobe, in which it has become more relevant to test hypotheses about arithmetic learning and speculate about the neural networks that support numerical cognition (Dehaene & Cohen, 1995).

According to the computer simulation model of how the brain can extract non-symbolic information for a visual set of objects (Dehaene & Changeux, 1993), the locations of each separate element are encoded by separate neurons. These neurons then transmit their information to a system that is tuned to a specific numerosity (for example, the number 7), and corresponds to said evidence, in the case of animals. In humans, similar types of brain pathways have been described for symbolic and non-symbolic numerosities (Santens et al., 2010). Interference has also been shown, for example, exhibited through longer reaction times, when the numerical size of the digits or their meaning is incompatible with the physical size of the font (Girelli et al., 2000; Henik & Tzelgov, 1982; Rubinsten et al., 2002).

In particular, the emergence of numerical automaticity appears to depend on formal education for symbolic numbers (e.g., numbers or words) (Girelli et al., 2000; Henik & Tzelgov, 1982; Rubinsten et al., 2002) and appears to be less dependent on formal education for non-symbolic numbers or numerosities, for example, represented as groups of dots (Gebuis et al., 2009).

The development of advanced mathematical skills later in life is linked to early proficiency in basic numerical activities like counting and numerical estimation (Libertus et al., 2013). According to Piazza et al. (2004), numerosities are represented less precisely as they increase, with larger numerosities being less precise than smaller ones, both behaviorally and neurally. Two consistent behavioral effects across languages and numerical notations (Buckley & Gillman, 1974; Dehaene, 1996) are, as follows:

1. The distance effect, where faster and more accurate responses occur when comparing numbers further apart (e.g., '2 versus 8' versus '2 versus 3'). However, this effect can reverse when symbolic numbers are correctly ordered, suggesting the importance of ordinality (Lyons & Beilock, 2013).
2. The size effect, where reaction times and error rates increase as the numerical magnitude of the numbers increases, regardless of the distance between them. For instance, comparing '2 versus 3' is faster than '7 versus 8' (Moyer & Landauer, 1967; Restle, 1970).

Therefore, this research goal is twofold:

(1) to articulate neuro-mathematics with the ETC that uses OSA tools and

(2) to explore the connections established by a teacher and his student when solving a problem about the volume of two boxes, one of toothpaste (made of cardboard) and the other of tomato (made of wood).

This work is important because in the teaching and learning processes of mathematics they must be simultaneously related to neuroscientific aspects, particularly, Alvarenga et al. (2022) maintains that there is a need to:

> Include aspects of neuroscience on the study of brain functioning in the act of learning mathematics as part of teacher training, including topics such as emotions, anxiety, gift, culture, brain plasticity among others with the purpose of integrating them into that training. Likewise, more collaborative research between these areas is necessary to obtain better dialogue and understanding between them to improve the quality of school teaching and learning (p. 1).

THREE THEORETICAL APPROACHES

Extended Theory of Connections Tools

In ETC theory, a mathematical connection is understood as a cognitive process and metaphorically as the tip of the iceberg made up of a conglomerate of practices, processes, PO identified in the mathematical activity of a subject when solving a task (intra or extra-mathematical) and semiotic functions (SFs) that relate them (Rodríguez-Nieto et al., 2022b). The connections are classified as intra-mathematical that "are established between concepts, procedures, theorems, arguments and mathematical representations among themselves" (Dolores-Flores & García-García, 2017, p. 160), and extra-mathematical where there is "a relationship of a mathematical concept or model with a problem in context (non-mathematical) or vice versa" (Dolores-Flores & García-García, 2017, p. 161). Additionally, the categories of mathematical connections are considered relevant:

1. *Modeling:* These are relationships between mathematics and real life and are evident when the subject solves non-mathematical or application problems where he or she has to propose a mathematical model or expression (Evitts, 2004).
2. *Procedural:* They arise when the subject uses rules, algorithms, or formulas to complete or solve a mathematical task. These mathematical connections are of form, A is a procedure used to work with concept B (García-García & Dolores-Flores, 2021).
3. *Different representations:* They are recognized when the subject represents a mathematical concept using alternative or equivalent representations (Businskas, 2008; García-García & Dolores-Flores, 2021). Equivalents are representation transformations performed in the same representation register. Alternate representations refer to representations where the register in which they were constructed is modified.
4. *Feature:* They are identified when the subject manifests characteristics of mathematical concepts or descriptions of their properties in terms of other concepts that make it different or similar to other concepts (Eli et al., 2011; García-García, 2019; García-García & Dolores-Flores, 2021).
5. *Reversibility:* It occurs when a subject starts from a concept A to reach a concept B and reverses the process starting from B to return to A (García-García & Dolores-Flores, 2021).
6. *Part-whole:* They manifest when the subject establishes logical relationships in two ways (general-particular and inclusion). The generalization relation is of the form A is a generalization of B, and B is a particular case of A (Businskas, 2008).
7. *Meaning:* It is identified when a subject "attributes a meaning to a mathematical concept, that is, what

Table 1. PO from the OSA perspective (Godino et al., 2019)

PO	Description
Problem situations/task (T)	Intra and extra-mathematical tasks, exercises, examples, questions, etc.
Linguistic elements (L-E)	Terms, expressions, notations, graphics, etc. in its different registers (written, oral or verbal, gestural, graphic, tabular, symbolic, etc.).
Concepts/definitions (D)	Introduced through definitions or descriptions, explicit or not (line, point, square, parallelepiped, area, volume, function, etc.)
Propositions/properties (Pr)	Statements or properties about concepts, etc.
Procedures (Pc)	Algorithms, operations, calculation techniques, using formulas, rules, etc.
Arguments (A)	They are statements used to validate or explain propositions and procedures, whether deductive or otherwise.

it means to him […]. It includes those in which a subject gives a definition that he has constructed for these concepts" (García-García & Dolores-Flores, 2021, p. 5). This connection is recognized when meaning is used to solve a problem.

8. *Implication:* They are identified when a concept *P* leads to another concept *Q* through a logical relationship ($P \rightarrow Q$) (Businskas, 2008). To identify this type of connections, the first linguistic form proposed by Selinski et al. (2014), who emphasized that "a student makes a connection with an explicit logical implication" (p. 559). In fact, logical implications were used to write if-then or to link words such as when, why, should, etc.
9. *Metaphorical:* They are understood as the projection of the properties, characteristics, etc., of a known domain to structure another less known domain (Rodríguez-Nieto et al., 2022c).
10. *Metaphorical connection based on mnemonics:* This connection is "understood as the relationship established by the subject between a mnemonic rule (often a familiar resource) and a mathematical object, rule, or mathematical procedure to memorize and use strategically more easily" (Rodríguez-Nieto et al., 2024, p. 18). These types of connections are inclusive and recursive where three elements must be considered:
    a. *keywords* that are similar to the word (or term) being referred to.
    b. *acronyms* it is identified when the first letter of each word is used in a list to construct another word.
    c. *acrostics* which consist of constructing a sentence, where the first letter of each constitutes the term studied (Rodríguez-Nieto et al., 2024).
11. *Instruction-oriented:* It refers to the understanding of a concept C based on two or more previous concepts A and B, required to be understood by a student. Furthermore, these connections can be identified in two ways:
    (a) the association of a new topic with previous knowledge and
    (b) the mathematical concepts and procedures connected to each other are considered prerequisites or skills that students must master before the development of a new concept (Businskas, 2008).

Furthermore, ETC has gained relevance and application in various current studies where they show theoretical-practical reflections (Font & Rodríguez-Nieto, 2024) and extra-mathematical connections in the context of a modeling problem (Ledezma et al., 2024).

Onto-Semiotic Approach Tools

The OSA is an inclusive and articulating theoretical system on mathematical knowledge and instruction, which was driven by the need to clarify and improve theoretical and methodological notions of different theoretical frameworks used in mathematics education from a unified perspective (Godino & Batanero, 1994).

According to Font et al. (2013) mathematical activity can be understood from an institutional or personal perspective, modeled in terms of practices and the configuration of PO and processes activated in said practices. Mathematical practice is "any situation or expression (verbal, graphic, tabular, symbolic) carried out by someone to solve mathematical problems, communicate the solution obtained to others, validate it or generalize it to other contexts and problems" (Godino & Batanero, 1994, p. 334). These practices involve PO understood in a broad sense as any entity involved in mathematical activity and that can be recognized as a unit (Font et al., 2013), for example: problems, notations, definitions, propositions, procedures, and arguments. The PO that emerge in mathematical practice can do so in different ways, which are the result of different ways of seeing, speaking, operating, etc. about PO; which allows us to talk about personal or institutional PO, ostensive or non-ostensive, unitary or systemic, intensive or extensive, and content or expression (Godino et al., 2007). In the OSA, six PO are considered (**Table 1**), which, related to each other, form configurations of PO (Godino et al., 2019).

A configuration is defined as the networks of primary or secondary objects that arise or intervene in systems of practices and are classified as epistemic (networks of institutional objects) or cognitive (networks of personal objects) (Godino et al., 2019). The epistemic configuration is the system of PO that, from an institutional perspective, are involved in the mathematical practices carried out to solve a specific problem and the cognitive configuration is the system of primary mathematical objects that a subject mobilizes as part of mathematical practices the subject develops to solve a specific problem (Godino et al., 2019).

The set of PO in **Table 1** emerges in mathematical activity through the activation of primary mathematical processes (communication, problem setting, definition, enunciation, elaboration of procedures and argumentation) derived from the application of the process-product perspective to these objects (Godino et al., 2007). These processes occur together with those derived from applying the process-product duality to the five dualities discussed above (institutional/personal, expression/content, ostensive/non-ostensive, unitary/systemic, and extensive/intensive): personalization-institutionalization; synthesis-analysis; representation-meaning; materialization-idealization; generalization-particularization (Font et al., 2013, 2016). For Godino et al. (2007), problem solving, and mathematical modeling should be called mathematical "hyper-processes" that combine some of the processes discussed.

In this theoretical approach, the SF is considered as the tool that allows practices to be related to the objects and processes that are activated and allows building an operational notion of knowledge, meaning, understanding and competence (Godino et al., 2007). A SF is a triadic relationship between an antecedent (initial expression/object) and a consequent (final content/object) established by a subject (person or institution) according to a certain criterion or correspondence code (Font, 2007). In the networking between the ETC and the OSA (Rodríguez-Nieto et al., 2022b), the SF is more general than the notion of mathematical connection, given that connections are considered particular cases of SFs of a personal or institutional nature. In ETC, the mathematical connection can be true or not, which is reinterpreted from the OSA in the following way: when a subject makes a correct connection it coincides with the institutional one and when it is incorrect it is only personal.

Neuro-Mathematics

Neuroeducation is a new discipline that is oriented towards "the teaching-learning process from the knowledge of applied neuroscience" (Bejar, 2014, p. 49). In that sense, this applied field of neuroscience implies relationships between education, pedagogy, didactics and neuroscience, therefore, its development seeks to empower the brain with respect to learning the different areas of knowledge and with it, to propose pedagogical strategies and didactics that guide learning and the environments that favor it. In this area of educational action, Neuro-didactics arises, which is established as an educational proposal of neuroscience through a focus on neurophysiology in mental processes, which allows for "effective, efficient, and timely teaching and learning strategies for attention to diversity and educational inclusion of all students" (Mora, 2007, p. 178).

The relationship between neuroscience and didactics in the area of mathematics is conducive to the emergence of a new discipline called neuro-mathematics. In this field of knowledge and action, cognitive neuroscience, psychology, neuro-didactics and mathematics didactics converge. In this sense, the following definition of *neuro-mathematics* is presented as:

> The scientific discipline that studies the application of knowledge and advances in neuroscience on the brain mechanisms associated with the learning of mathematics and the pedagogical and didactic processes given in the teaching and learning of mathematics (Giraldo et al., 2021, p. 380).

Neuro-mathematics seeks to understand the cognitive, emotional, and operational processes of the mind when learning mathematics, in which behavioral and neuroscientific methods are also combined with the aim of achieving a broader understanding of neurocognitive mechanisms that underlie learning and to support the development of effective instruction in mathematics.

In this line of thought, Leikin (2018) presented the cognitive mechanisms of higher mathematics, applying different types of complex mathematical problems, and explored brain activation patterns in students with different mathematical abilities and in generally gifted students. The results appear to support the hypothesis that efficient mathematical problem solving is related to an overall decrease rather than an increase in brain activation. The above results generated discussion among groups of theorists about the potential and limitations of the integration of neuroscience in mathematics education research. The above makes it interesting to approach neuroscience in the field of mathematics, in order to achieve a greater understanding of the intuitive mechanisms contained in the resolution of mathematical problems, or of the cognitive and emotional aspects that underlie said mental process.

For the purposes of this research, especially in the analysis and results sections of neuro-mathematical connections, important meanings of neuro-mathematics and neuroscience are required. For example, recognition of terms and symbols is understood as the multimodal linguistic skill that refers to the understanding, combination and use of the multiple ways that exist for people to communicate and express themselves (gestures, images, technologies and communication) beyond of spoken language to promote understanding in mathematics teaching (Banegas, 2023).

Visual perception, spatial skills and motor coordination are three abilities that are closely related, playing a crucial role in the learning and academic performance of students, because visual perception is the ability of human beings to process and understand visual information from the environment. Spatial skills are related to the ability to process and understand the position, orientation and movement of objects in space, which are evident in tasks such as reading and writing and, finally, fine motor coordination is involved in the ability that the human being has to coordinate the movements of the hands and eyes to perform tasks that require precision, for example writing, drawing (Gutiérrez & Neuta, 2015; Macías & Cuellar, 2018; Narváez-Rumié et al., 2019 ; Price & Henao, 2011).

The *association of concepts and mathematical formulas* is defined as the ability to link abstract ideas with mathematical expressions that represent them, allowing us to describe and analyze phenomena in a quantitative and precise way, enabling the union between ideas, variables and values to solve problems and describe relationships organized and systematic manner (Coronado, 1998; Godino et al., 2003; Mora, 2003). *Intermediate calculations and unit conversions* are mathematical operations performed in a process of solving a problem or equation, which may involve unit conversions, arithmetic operations, and applications of mathematical formulas (Arfken, 1985; Serway, 1990). These intermediate calculations allow for a deeper understanding of the concepts and obtaining accurate results.

For its part, solving operations step by step and understanding the process consists of breaking down complex mathematical problems into simple steps in order to facilitate the understanding and effective resolution of the proposed problem or equation. Specifically, Mayer and Moreno (2003) support the effectiveness of this strategy by demonstrating that it reduces cognitive load and improves understanding of mathematical concepts. In relation to verification, it was found that it involves the process of checking or confirming that a statement or solution to an operation or problem is correct or incorrect (Azcárate & Camacho, 2003), and the conclusion is the final result of a process which is arrived at after following logical and systematic reasoning based on established premises, definitions and theorems (Lakatos, 2015).

METHODOLOGY

This research is qualitative exploratory (Cohen et al., 2018) that seeks to analyze the neuro-mathematical and mathematical connections of a teacher and his student in solving geometric problems, developed in two stages:

1. The theories are articulated considering the agreements and complementarities highlighting the notion of mathematical connection and the creation of neuro-mathematical connection.
2. A reflection context developed in three phases was considered to apply theoretical integration: the participants were selected, a non-participant observation was carried out in the classroom in the resolution of a problem and then the data were analyzed with the new theoretical vision and the category of neuro-mathematical connection, which, in fact, is the analysis of the results.

Strategies for Networking of Theories

To achieve theoretical articulation, the pairs of strategies to articulate theories proposed by Prediger et al. (2008), which range from completely ignoring another theoretical framework at one extreme, to globally or partially unifying different approaches at the other. Also, there are intermediate strategies presented in a hierarchical manner, therefore, the first two pairs of strategies refer to the understanding of both theories by the experts of each theory or theoretical approach. The second pair of strategies involves a comparison and contrast of the theories to identify different commonalities or differences between the theories. The third pair guides researchers to the combination and coordination of theories, leaving a framework of conceptual complementarities to generate a new theory or methodology. In the fourth pair of strategies, complementarities are locally integrated and synthesized, leading to the formation of a holistic theoretical framework.

Understanding of theories

In this article it is considered that the first and second pair of networking strategies have been developed in the communication and understanding of the theoretical approaches presented before, where the principles and methods of each theoretical approach are evident. However, in the following sections some of these aspects are explained because to carry out the comparison the perspective of Radford (2008) was followed, who stated that in networking between theories differences and complementarities should be considered based on principles, methodologies, and theory research questions.

Comparing and contrasting theories

In the literature on neuroscience, neuroimaging, neuropsychology, neuroeducation, among others, the connections in the brain have been recognized, which occur between neurons and are called synapses, for example, neurons are connected to others by dendrites (incoming connections) and axons (outgoing and fast connections). These types of connections are activated in the brain areas depending on the function that the human being is going to perform, specifically the connection occurs in drill and Wernicke when it comes to language production and understanding. In this way, other connections are encouraged in the occipital lobe when it activates the visual and identification and graphics areas; Connections established when performing mathematical calculations and are found in the parietal lobe, temporal lobe, and prefrontal cortex.

While, in the theory of mathematical connections, connections are understood as a cognitive process where the subject relates procedures, theorems, representations, concepts, meanings, through SFs that are undoubtedly important for the understanding of mathematical concepts. These types of connections depend on what happens in people's brains, for example, to establish connections, relationships between neurons are activated that allow the human being to visualize, represent, execute mathematical calculations, etc.

In this context, it follows that both theoretical positions have the term connection in common and consider it relevant to all the activities that human beings carry out in their daily lives. However, mathematical connections differ from neural connections because the former have been explored in a paper-and-pencil environment, have been inferred from videos, written productions, and the latter are observed in functional magnetic resonance neuroimaging studies through Electroencephalogram as a technique to study neuronal activity and functioning. It should be noted that neural connections are analyzed in some research based on previous validated studies where each human action represents one or more neural connections.

From the methodological processes, the mathematical connections have been explored from an exploratory qualitative perspective using thematic analysis methods or the analysis of practices, processes, objects, and establishment of SFs proposed in the OSA. While, in neuroscience, qualitative (Alvarenga et al., 2022; Mendoza-Arenas et al., 2022), quantitative (Sagula, 2023) and literature review research has been carried out, for example, Calzadilla-Pérez (2023) made a scientometric mapping of the neurosciences of education with a view to contributing to teacher training.

In relation to the research questions that some authors have asked from the two theoretical approaches, they are linked to both neural and mathematical connections. For example, in neuro-mathematics Vargas-Vargas (2013) delved into the links between mathematics and neuroscience, identifying relevant questions to delve deeper into this field, among which the following stand out: are mathematical concepts innate or are they learned? If they are learned, when are

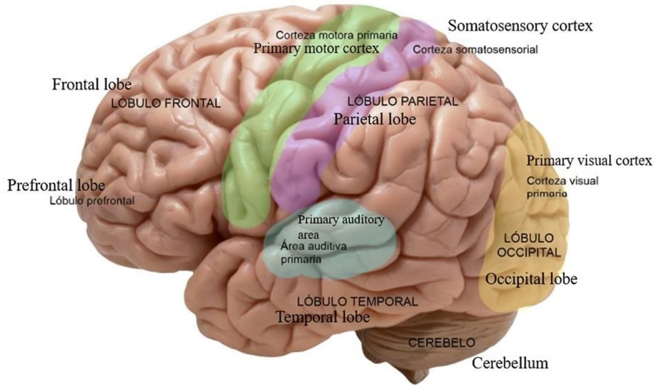

Figure 1. Brain areas involved in mathematics processing (Vargas-Vargas, 2013)

they learned? What areas of the brain are in charge of the mathematical task? Which have motivated work related to the functioning of the brain environment according to the mathematical abilities of the subjects, as well as the areas involved in mathematics and its brain processing: primary motor, somatosensory, visual, and auditory cortices (see **Figure 1**).

In Rodríguez-Nieto et al. (2022b) it is evidenced that the research carried out under the ETC framework have answered research questions such as the following: What connections are promoted when studying a specific mathematical object? What is the level of quality of the mathematical connections established by students or teachers? What factors must be present for a new typology of mathematical connections to be generated? What are the connections the teacher makes in the classroom? What mathematical connections are presented in school mathematics textbooks, and which are promoted in the curricula and curricula of the different countries of the world? How could teaching interventions be developed that help promote connections and develop in students the ability to use mathematical connections in different mathematical and extra-mathematical domains? What are the beliefs that both students and teachers attribute to the use and importance of mathematical connections and what is your perception of the role they play in teaching-learning? (García-García, 2019; García-García & Dolores-Flores, 2018; 2021; Rodríguez-Nieto et al., 2023). Also, the ETC is interested in exploring mathematical and ethnomathematical connections in cultural groups, considering ethnomodeling (Rodríguez-Nieto et al., 2023a) and mathematical modeling. In addition, it is necessary to build and validate a frame of reference to study mathematical understanding from connections (García-García, 2019).

Coordination and combination of theories

It was identified that the two theoretical approaches juxtapose each other and lead to establishing common points of view, showing that they complement each other. A particular case is that ETC has been investigated

Table 2. Data analysis method from OSA-ETC and neuro-mathematics integration

Phases	Description
1 Transcription of interviews/observations or organization of written productions	The students' written productions were organized where they were given labels of the participants P1 (student) and P2 (teacher) and likewise, the researchers became familiar with the participants' responses. Additionally, this process is essential to ensure that the researcher thoroughly reads, analyzes, and understands the information collected.
2 Temporal narrative	The student's resolution of the problem is explained mathematically. In turn, it contains the practices carried out by the student and some important primary objects identified in the narrative that are considered the protagonists of the mathematical activity. This narrative is used for the analysis of neuro-mathematical and mathematical connections.
3 Mathematical practice	Mathematical practices are described as a sequence of actions regulated by institutionally established rules useful for solving problems. In these mathematical practices, the foundation of each connection in the brain is evident from neuro-mathematics and on paper from the ETC articulated with the OSA. Basically, the identification of mathematical practices depends on coding (c), in other words, it would be the raw data acquired from the transcription of the speech in participant observation.
4 Cognitive configuration	System of primary mathematical objects that a subject mobilizes as part of the mathematical practices developed to solve a problem. These primary objects are a fundamental part of the connection because they are generally the beginning (antecedent) and the end (consequent) of its structure.
5 SF	are established between the primary objects of the cognitive configuration. In this way, the mathematical connections proposed by the ETC are formed and visualized.
6 Mathematical connections	The practices, processes, objects, and SF that relate them are captured in a table to give origin and detail to the mathematical connection.
7 Neuro-mathematical connections	For each of the mathematical practices there are implicitly neuro-mathematical connections that relate the brain areas before the mathematical connections are visualized or externalized.

based on the analysis of problem-solving processes on mathematical objects such as derivatives, addition and subtraction, functions, geometric solids, etc., but it does not emphasize the connections that are activated in people's minds. In this line, there is a fundamental complementarity because neuro-mathematics deals with the mathematical activity of the subject in his mental structure.

Now, in the first pairs of strategies the approaches were understood, differences and similarities were found between them, but in the pair of coordination and combination strategies, in addition to finding complementarities, Bikner-Ahsbahs and Prediger (2010) affirm that the articulation should not be complete only between theories but are used "primarily for a networked understanding of an empirical phenomenon or a piece of theoretical data" (p. 10). Therefore, the analysis of a phenomenon regarding the resolution of a geometry problem based on neuro-mathematical and mathematical connections is presented below. Likewise, the analysis of the phenomena ensures the functioning of the theoretical tools articulated as evidenced in Font et al. (2016) on the notion of object from the APOS and the OSA, Hummes et al. (2022) delving into the lesson study from the OSA, Ledezma et al. (2022) articulated OSA tools to improve practical argumentation in modelling contexts, Ledezma et al. (2023) articulating the modeling with the OSA, Bikner-Ahsbahs (2022) worked on covariational reasoning and argumentation, Rodríguez-Nieto et al. (2023a) used the networking between the ETC and the OSA to analyze the derivative graph, among others.

On the other hand, the analysis methods of these two theories are common and the analysis method used in the ETC was practically used, nourished with the OSA tools that allow a more detailed analysis of the mathematical activity (see **Table 2**).

Context of Reflection on Geometry

A reflection context developed in three phases was considered to apply networking of theories to a phenomenon (Bikner-Ahsbahs & Prediger, 2010): selection of participants; carrying out non-participant observation in the classroom in the resolution of a problem and then, the data were analyzed with the new theoretical methodological view, which, in fact, is the analysis of the results.

Study context and participants

This research was carried out in a public institution in the municipality of Sabanalarga, Atlántico, Colombia, and a twenty-nine-year-old high school mathematics teacher (P2) with seven years of work experience, and an eleventh-grade student (P1) who was chosen, participated by the teacher to explain the resolution of a problem about the volume of a box to an eighth-grade class in the subject of geometry. It should be noted that this student had developed the topic of volumes of geometric solids in previous courses and had an average of 9.5 out of 10 in the three academic periods that he took during the year.

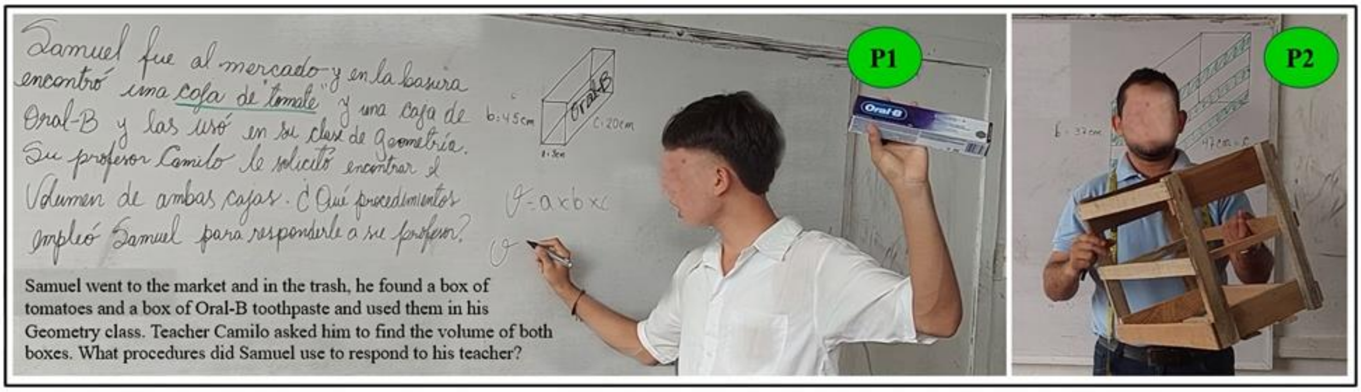

Figure 2. Evidence from participant observation (Source: Authors' own elaboration)

Data collection

The data were collected by the authors of this research, who requested permission from the coordination of the public educational institution to record the class, which was granted because the first author of this article was a teacher at said institution. After agreeing on the participations, participant observation was carried out (Cohen et al., 2018) of the class where an episode was selected where the student and the teacher explained the resolution of a problem as described below and in **Figure 2**:

Problem proposed to the student and the teacher: Samuel went to the market and in the trash he found a box of tomatoes and a box of Oral-B toothpaste and used them in his geometry class. Teacher Camilo asked him to find the volume of both boxes. What procedures did Samuel use to respond to his teacher?

> Measurements of the Oral-B box (toothpaste): width a = 3 cm; height b = 4.5 cm and length c = 20 cm; measurements of the tomato box: width a = 28 cm; height b = 37 cm and length c = 47 cm.

It should be noted that the problem practically consists of two parts and student P1 decided to solve the second part that tries to find the volume of the oral-B box (toothpaste) and then, teacher P2 stated that he found the volume of the tomato box.

Data analysis

To analyze the data, the integrative method proposed in **Table 2** is used, highlighting those phases 1, 2, and 3 will be used in a similar way for both analyzes for the ETC-OSA and neuro-mathematics. In this sense, the results section shows the complete and detailed analysis of the mathematical activity of the teacher and the student.

FINDINGS

This section shows the neuro-mathematical and mathematical connections activated by the student and the teacher in solving geometry problems.

Temporal Narratives of the Student and Teacher

Student's narrative

The teacher (P2) proposed a problem to find the volume of a box, which was read and understood by P1. Then, P1 draws the box on the board considering its measurements in centimeters and in the shape of a parallelepiped. Subsequently, to find the volume P1 consider the formula $v = a * b * c$ and assume that a is the base (width) equal to 3cm, b is equal to the height b=4.5 cm and c is equal to 20 cm which is the depth. P1 uses the associative property and multiplies $3cm * 4.5cm$ obtaining as a result $13.5cm^2$ and, finally, multiply $13.5cm^2$ by 20 cm to get the volume of the box equal to $270\ cm^3$.

Teacher's narrative

The problem was proposed by P2 who read and understood the problem and told the students that they were going to work with the volume of the tomato box that can be found considering the following measurements. Then, P2 drew the box on the board according to the established measurements and with the shape of a parallelepiped. Later, he mentioned that they should apply the volume formula equal to $v = a * b * c$. Next, he substituted the values of a, b, and c into the formula $v = (28cm)(37cm)(47cm)$ and multiplied it considering the associative property $28cm * 37cm$ resulting 1.036 cm^2 and he multiplied that value by 47 cm, obtaining 48.692 cm^3. Finally, P2 stated that 48.692 cm^3 is the volume of the tomato box and the parallelepiped simultaneously.

Mathematical Practices System

P1 mathematical practices (P1mp)

P1mp1. P1 read and understood the proposed problem and mentioned what it represents a parallelepiped (C1).

P1mp2. P1 graph the Oral-B box (toothpaste) assuming its dimensions and the shape of a parallelepiped with rectangular faces (C2).

P1mp3. P1 considered the formula $v = a * b * c$ and assumed that a is the base equal to 3cm, b is equal to the height $b = 4.5$ cm and c is equal to 20 cm which is the depth (C3).

P1mp4. P1 used arithmetic operations and properties such as associative to multiply 3cm*4.5cm and got $13.5cm^2$ (C4).

P1mp5. P1 multiplied $13.5cm^2$ by 20 cm to get the volume of the box equal to $270\ cm^3$ (C5).

P1mp6. P1 verified the procedures carried out (C6).

P2 mathematical practices (P2mp)

P2mp1. P2 read and understood the problem he proposed and mentioned that it represents a parallelepiped (C7).

P2mp2. P2 graph the tomato box assuming its measurements and parallelepiped shape (C8).

P2mp3. P2 considered the formula $v = a * b * c$ and substitutes the values of a, b and c in the formula $v = (28cm)(37cm)(47cm)$ (C9).

P2mp4. P2 used the associative property to multiply $28cm * 37cm$ and got $1.036\ cm^2$ (C10).

P2mp5. P2 multiplied $1.036\ cm^2$ by $47\ cm$ and got $48.692\ cm^3$ (C11).

P2mp6. P2 assumed that $48.692\ cm^3$ is the volume of the tomato box and the parallelepiped simultaneously (C12).

Cognitive Configuration of Primary Objects

The sequenced actions carried out by P1 to solve the problem posed were evident. In mathematical practices, PO emerge that, connected to each other, make up networks of personal or cognitive objects (see **Table 3**).

Table 3. Cognitive configuration of primary objects of P1 and P2

PO	P	Description
T	P1 and P2	Samuel went to the market and in the trash he found a box of tomatoes and a box of Oral-B toothpaste and used them in his geometry class. Professor Camilo asked him to find the volume of both boxes (measurements of the Oral-B box (toothpaste): width a = 3 cm; height b = 4.5 cm and length c = 20 cm; measurements of the tomato box: width a = 28 cm; height b = 37 cm and length c = 47 cm). What procedures did Samuel use to respond to his teacher?
L-E	P1	*Verbal:* Rectangle, edge, parallelepiped, angle, volume, measurement, centimeter, multiplication...
		Symbolic: Measurements for the Oral-B box (toothpaste) ($a = 3\ cm, b = 4.5\ cm, c = 20\ cm$). $v = a * b * c$; $3\ cm * 4.5\ cm$; $13.5\ cm^2$; $13.5\ cm^2 * 20\ cm$; $270\ cm^3$.
		Graphic: Representation of the toothpaste box (**Figure 3**).

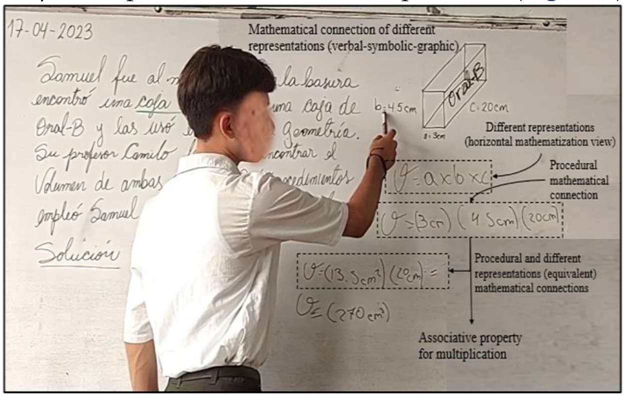

Figure 3. Graphic representation of the oral-B box with a parallelepiped shape by P1 (Source: Authors' own elaboration)

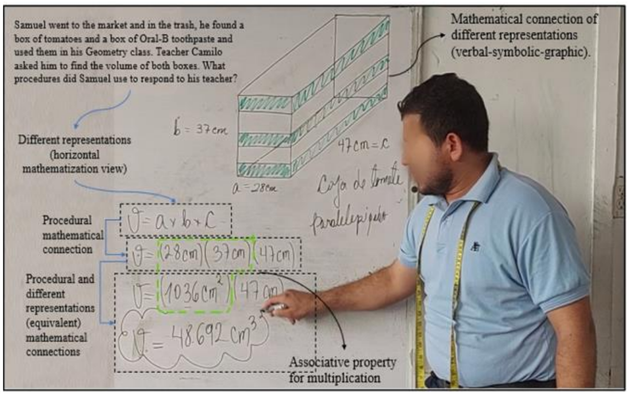

Figure 4. Graphic representation of the parallelepiped-shaped tomato box by P2 (Source: Authors' own elaboration)

PO	P	Description
	P2	*Verbal:* Rectangle, edge, parallelepiped, angle, volume, measurement, centimeter, multiplication...
		Symbolic: Measurements for the tomato box ($a = 28\ cm, b = 37\ cm, c = 47\ cm$); $v = a * b * c$; $3\ cm * 4.5\ cm$; $13.5\ cm^2$; $13.5\ cm^2 * 20\ cm$; $270\ cm^3$.
		Graphic: Representation of the tomato box (**Figure 4**).
D	P1 and P2	*Previous concepts/definitions:* Rectangle, edge, parallelepiped, angle, volume, measurement, centimeter, multiplication ...
		Definition 1 (D1): Volume is a magnitude defined as the place a body occupies in space and is expressed in cubic units.
		D2: A parallelepiped is a polyhedron or prism made up of six faces (hexahedron), twelve edges, eight vertices and the faces are parallelograms (squares, rectangles, rhombuses or rhomboids). In addition, there are five types of parallelepipeds (straight, oblique, orthohedron, cube and rhombohedron).
Pr	P1	*Pr1:* The box is shaped like a parallelepiped as drawn on the board.
		Pr2: The formula or model to find the volume of the box is $v = a * b * c$.
		Pr3: The volume of the oral-B box is $270\ cm^3$.

Table 3 (Continued). Cognitive configuration of primary objects of P1 and P2

PO	P	Description
	P2	*Pr1:* The box is shaped like a parallelepiped as drawn on the board. *Pr2:* The model to find the volume of the box is $v = a * b * c$. *Pr3:* The volume of the tomato box and the parallelepiped is 48.692 cm^3.
Pc	P1	*Main procedure of P1 (MpcP1):* Find the volume of the oral-B box (toothpaste). *Auxiliar procedure (Apc1):* P1 draws the box in the shape of a parallelepiped and assigns measurements to it. *Apc2:* P1 applies the formula $v = a * b * c$. *Apc3:* P1 applies the associative property for the multiplication of 3 cm * 4.5 cm = 13.5 cm^2. *Apc4:* P1 multiplies 13.5 cm^2 * 20 cm and it turns out 270 cm^3.
	P2	*Mpc of P2:* Find the volume of the tomato box. *Apc1:* P2 draws the box in the shape of a parallelepiped and assigns measurements. *Apc2:* P2 applies the formula $v = a * b * c$. *Apc3:* P2 applies the associative property for multiplication of 28 cm * 27cm = 1.036 cm^2. *Apc3:* P2 multiplies 1.036 cm^2 * 47 cm and it turns out 48.692 cm^3.
A	P1	Argument of P1 (A1) *Thesis:* The oral-B box represents a parallelepiped. *Reason 1 (R1):* The oral-B box has six faces and all of them are rectangles including their bases, that is, it is shaped like a parallelepiped. *Conclusion:* The oral-B box is a parallelepiped. A2 of P1 *Thesis:* The volume of the oral-B box is 270 cm^3. *R1:* P1 uses the formula $v = a * b * c$ to find the volume which is generally multiplying the area of the base by the height of the prism. *R2:* P1 applies the associative property to execute the multiplications 3cm * 4.5cm * 20cm equal to 270 cm^3. *Conclusion:* The volume of the oral-B box is 270 cm^3.
	P2	A1 of P2 *Thesis:* The tomato box represents a parallelepiped. *R1:* The tomato box has six faces, and they are all rectangles including their bases, that is, it is shaped like a parallelepiped. *Conclusion:* The tomato box is a parallelepiped. A2 of P2 *Thesis:* The volume of the tomato box is 48.692 cm^3. *R1:* Use the formula $v = a * b * c$ to find the volume, which is generally multiplying the area of the base by the height of the prism. *R2:* Applies the associative property to perform multiplications 28cm * 27cm * 47cm equal to 48.692 cm^3. *Conclusion:* The volume of the tomato box is 48.692 cm^3.

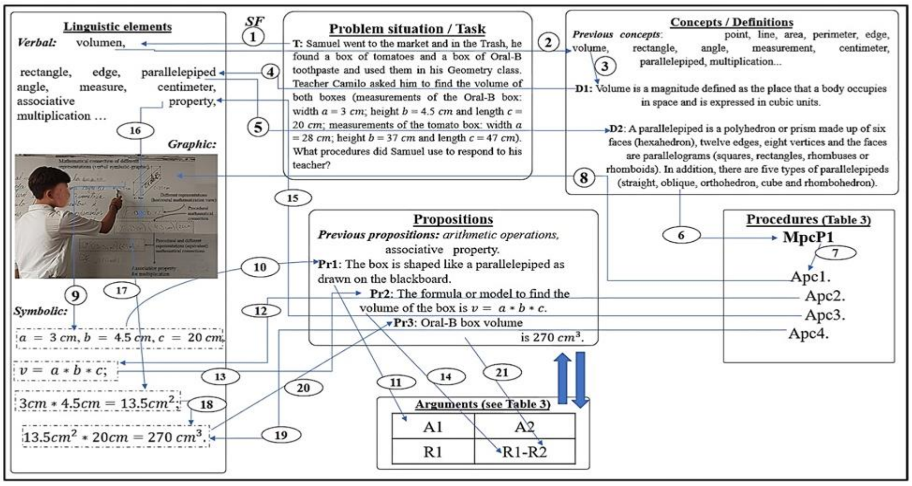

Figure 5. SFs of P1 (Source: Authors' own elaboration)

Semiotic Functions

After writing the systems of practices and constructing the cognitive configurations of P1 and P2, **Figure 5** and **Figure 6** present the SFs that relate the PO.

Now, the cognitive configurations of P1 and P2 reveal similarities in the procedures used to solve the problem as agreed upon in class. It is known that the boxes have different measurements, but they represent the same

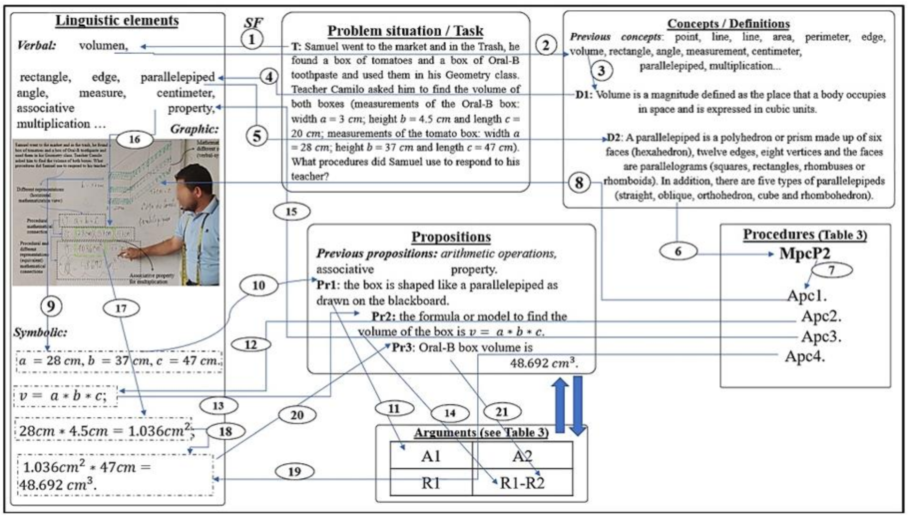

Figure 6. SFs of P2 (Source: Authors' own elaboration)

Table 4. Structure and functioning of mathematical connections

Mp	Processes	Hyper-processes	Objects	SFs	Mathematical connections (ETC)
Mp1	-Signification/ understanding -Problematization -Enunciation	-Problem-solving	P1 and P2 read the problem and give meaning and use to the concept of volume and parallelepiped.	SF1, SF2, SF3, SF4, & SF5	Meaning & feature
Mp2	-Enunciation -Idealization -Representation -Particularization	-Problem-solving	P1 and P2 graphically represented the parallelepiped on the board associating measurements with it.	SF6, SF7, SF8, SF9, SF10, & SF11	Feature, different representations (alternate), procedural, & part-whole
Mp3	-Representation	Problem-solving	P1 and P2 used the model or formula $v = a * b * c$ and associated the measurements to find the volume.	SF12, SF13, & SF14	Feature, different representations (horizontal mathematization view), & procedural
Mp4	-Representation -Algoritmization	-Problem-solving	P2 used the associative property to multiply $28cm * 37cm$ to get $1.036\ cm^2$. P1 did the same procedure.	SF15, SF16, & SF17	Part-whole, different representations (equivalent), & feature
Mp5	-Algoritmization -Representation -Argumentation	-Problem-solving	P2 multiplies 1,036 cm^2 by 47 cm and obtains 48,692 cm^3. P1 did the same procedure.	SF18, SF19, SF20, & SF21	Procedural & different representations (equivalent)
Mp6	-Argumentation -Signification	-Problem-solving	P2 assumed that 48.692 cm^3 is the volume of the tomato box and the parallelepiped simultaneously. For his part, P1 verified the procedures to ensure that the operations are well done.	SF21	Meaning

type of parallelepiped, therefore, in **Table 4** the mathematical connections are presented once and equally for P1 and P2. From **Table 4**, the synthesis and local integration of theories is established where the theoretical elements are used or related deeply for data analysis.

The analysis of the mathematical connections with the ETC carried out previously requires the mathematical practices, processes, objects, and SFs that are tools of the OSA (see Rodríguez-Nieto et al., 2022b) that, without a doubt, qualify said analysis and show the constitution of the connections. However, with the new articulation between ETC and neuro-mathematics, the analysis can be delved deeper because the neural connections that occur in the different areas of the brain associated with the mathematical processes that the person does are recognized (P1 and P2) when they solve the proposed problem. Now, in the new analysis it is considered that for the development of each mathematical practice there have been neuro-mathematical connections as shown below.

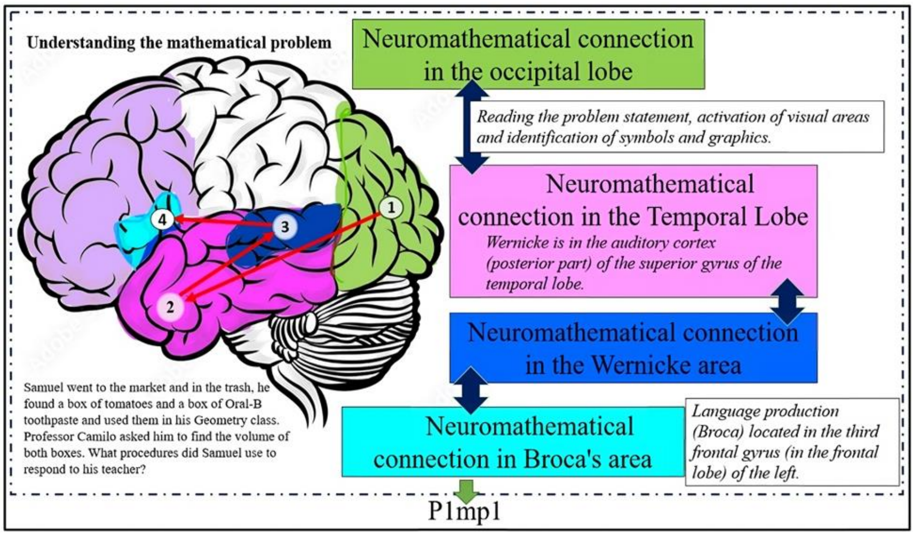

Figure 7. Neuro-mathematical connections for P1mp1 (Source: Authors' own elaboration)

It should be noted that, in Mp3 of **Table 4**, a connection of different representations is evident, but with a perspective of horizontal mathematization (Freudenthal, 1991) where P1 and P2 transitioned from the verbal language of the problem situation to the symbolic language, which, in fact, it can be called a normative model to calculate the volume of the box (Blum, 2002).

Neuro-Mathematical Connections

These types of connections were identified considering the theoretical and applied results evidenced in the literature on neuroscience and neuro-mathematics (De la Serna, 2020; Giraldo-Rojas et al., 2021; Guerrero, 2021; Verschaffel et al., 2016), which explains the activation of neural connections associated with mathematics (neuro-mathematical connections). It should be noted that these types of connections are hypothetical, that is, those that according to theory or literature are being activated for each mathematical practice.

Neuro-mathematical connection of term and symbol recognition (associated with P1mp1)

When the student reads the statement, the main areas of the brain that are activated are the visual ones, which are located in the occipital and temporal lobes, whose function is to participate in visual perception and the identification of symbols and words. Likewise, the areas of the brain involved in language comprehension (Wernicke) which is located behind the primary auditory cortex, in the posterior part of the superior gyrus of the temporal lobe, and language production (Broca) located in the third gyrus frontal (in the frontal lobe) of the left, in some exceptional cases it is found in the right hemisphere, which are connected by a bundle of nerve fibers called the arcuate fasciculus in order to process the meaning of words and the relationship between them, That is, the student attributes meanings and characteristics to the box in the manner of a parallelepiped (see **Figure 7**).

Neuro-mathematical connections of visual perception, spatial skills, and motor coordination (P1mp2)

When the student proceeds to draw the parallelepiped, the connections that are activated are those linked to the visual, spatial, and motor process, which is why visual perception is primarily activated to identify the shape and dimensions that the parallelepiped should have. This process begins in the primary visual cortex which is in the posterior pole of the occipital lobe, in which visual signals from the environment are processed. Likewise, the fusiform gyrus also plays a fundamental role since it is responsible for recognition of three-dimensional shapes and give them meaning, spatial memory, located in the hippocampus, also plays an important role in solving the problem, since the student must remember and imagine the parts and shape of the parallelepiped. Then, when drawing the parallelepiped, the motor cortex, which is in the frontal lobe, and the cerebellum located at the back and bottom of the skull, come into play, because fine motor skills are involved in coordination of the hand and fingers when drawing the lines and shape of the parallelepiped (**Figure 8**).

Neuro-mathematical connection related to the association of mathematical concepts and formulas (P1mp3)

Regarding symbolic representation, it could be said that the integration areas in the frontal and parietal lobe work together to facilitate the understanding of the relationship between the values a, b, and c and how these are related to volume. In turn, areas such as the horizontal segment of the intraparietal sulcus and the angular gyrus associated with numerical and

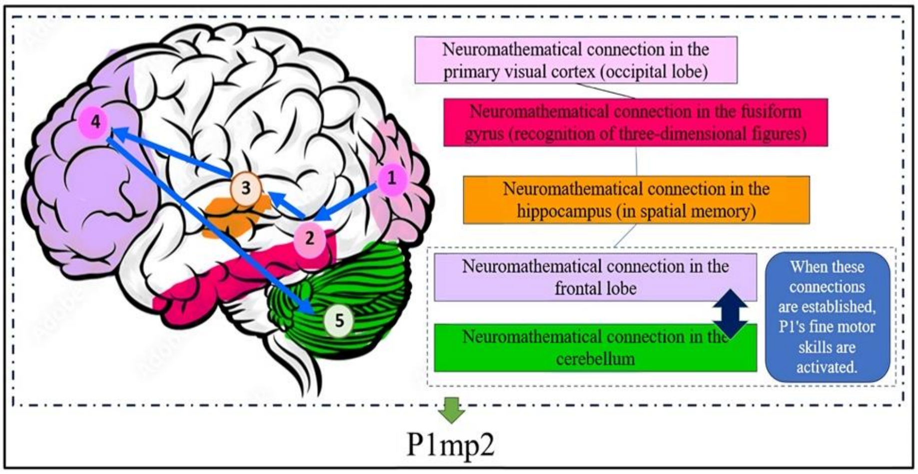

Figure 8. Neuro-mathematical connections for P1mp2 (Source: Authors' own elaboration)

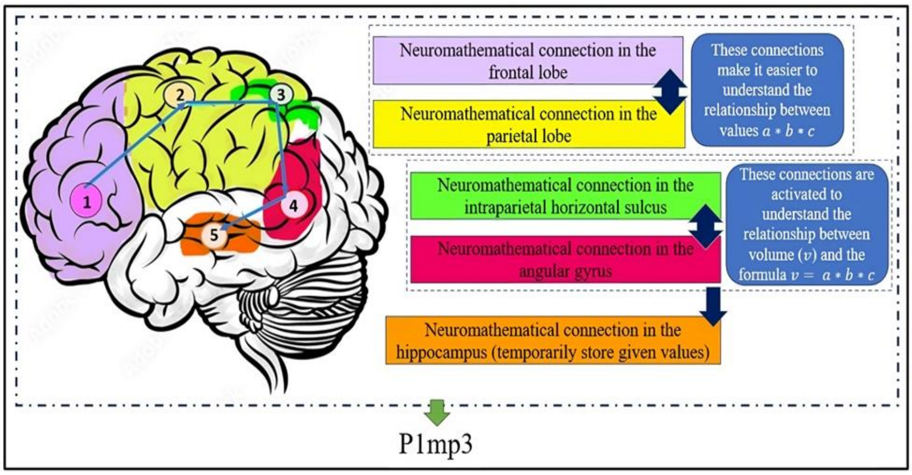

Figure 9. Neuro-mathematical connections for P1mp3 (Source: Authors' own elaboration)

mathematical processing are activated to understand the relationship between volume (v) and the formula $v = a * b * c$; here the brain identifies that "$a = 3\ cm$" represents the base or width, "$b = 4.5\ cm$" represents the height "$c = 20\ cm$" represents the depth or length.

The student uses short-term memory, which is in the hippocampus and its function is to temporarily store the values given in this case, to each of the letters and the relationship given between them in order to calculate the volume (**Figure 9**).

Neuro-mathematical connection of intermediate calculations and unit conversion (P1mp4)

Now the student proceeds to multiply the values $3\ cm * 4.5\ cm$, the neural connections associated with mathematical calculation, which are in the parietal lobe, temporal lobe and prefrontal cortex, are activated to execute the operation and thus the student obtains $13.5\ cm^2$ as a result. After this, the working memories located mainly in the frontal lobe of the brain will keep the previously obtained result ($13.5\ cm^2$) stored partially while the missing operation is performed (**Figure 10**).

Neuro-mathematical connection of solving operations step by step and understanding the process (P1mp5)

Before the student proceeds to solve the final operation, the brain plans the next step to be carried out, at this point the prefrontal areas, especially the dorsolateral cortex of the brain, can be activated in the integration of intermediate calculations ($13.5\ cm^2 * 20\ cm$), when performing the multiplicative operation, the brain areas related to mathematical calculation return to the ring again, the brain executes the operation and the student obtains $270\ cm^3$ as a result (**Figure 11**).

Neuro-mathematical connection of verification and conclusion (P1mp6)

Finally, when solving the problem, several areas responsible for verifying and integrating information such as the prefrontal cortex, the parietal cortex, the temporal cortex, the ventromedial prefrontal cortex, the dorsolateral prefrontal cortex, the hippocampus, the thalamus and the corpus callosum which although not as such a specific area is crucial since it allows the connection between the cerebral hemispheres that work together to ensure that the steps taken by the student to

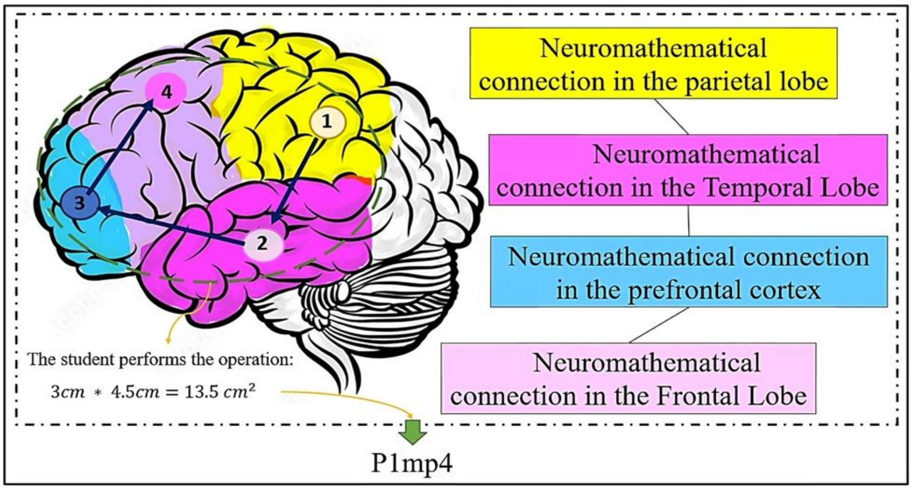

Figure 10. Neuro-mathematical connections for P1mp4 (Source: Authors' own elaboration)

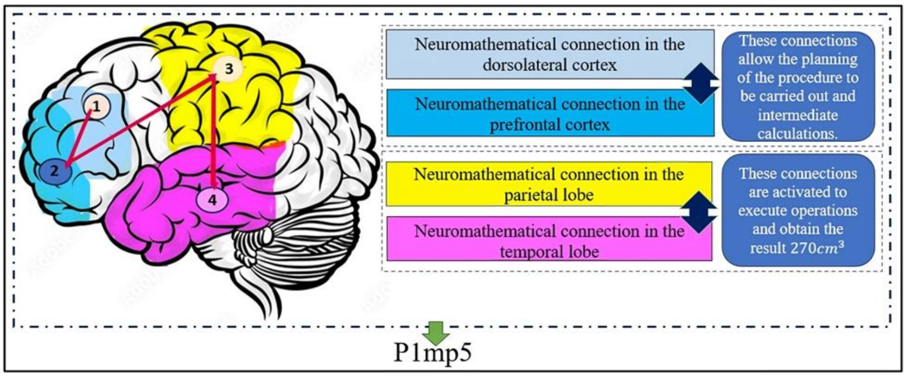

Figure 11. Neuro-mathematical connections for P1mp5 (Source: Authors' own elaboration)

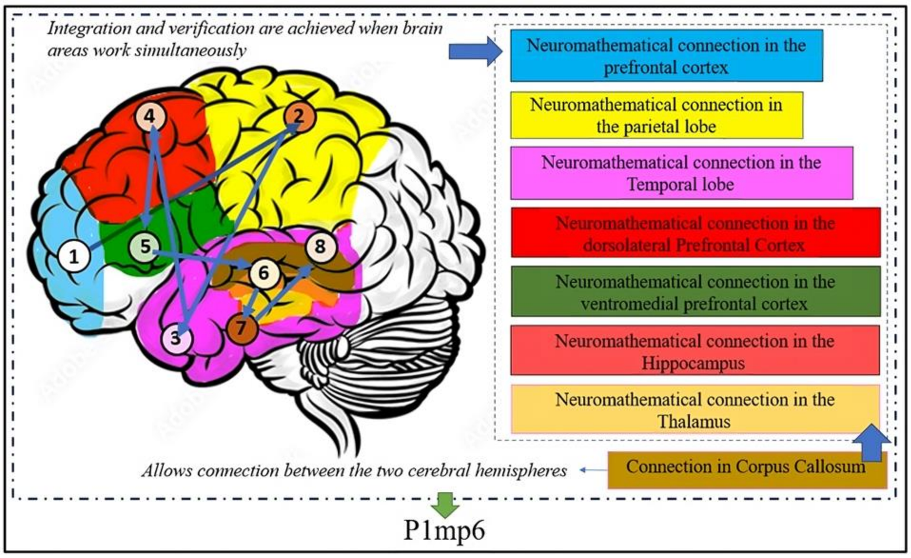

Figure 12. Neuro-mathematical connections for P1mp6 (Source: Authors' own elaboration)

solve the problem have been taken in the correct way and that the result obtained is consistent with the operations carried out (**Figure 12**).

DISCUSSION

In this research, a new networking of theories was developed between the ETC (which had already been combined with the OSA) and neuro-mathematics that merits the use of neural connections associated with the procedures and use of mathematical laws. Likewise, another way to deepen and/or detail the multiple mathematical connections that people make when they solve mathematical problems is shown, leaving open a path of exploration towards new research on connections that address brain areas.

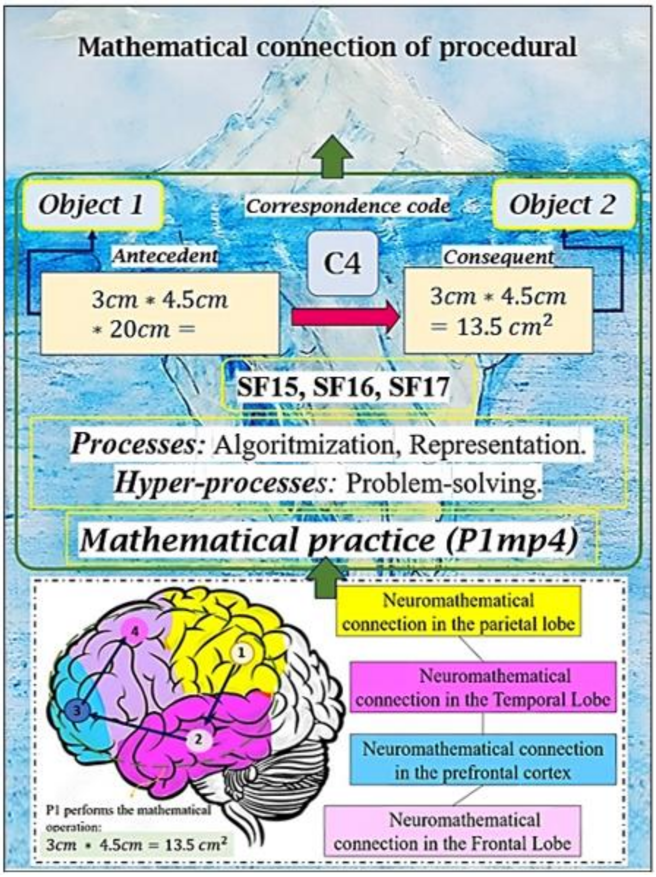

Figure 13. New view of mathematical connection as an iceberg (Source: Authors' own elaboration)

We revealed that the ETC-OSA networking required more depth to analyze phenomena and has been achieved with the networking with neuro-mathematics. On the one hand, each of the mathematical connections can be explained not only as they occur externally with pencil and paper or marker and board (Campo-Meneses & García-García, 2023; De Gamboa et al., 2023; García-García & Dolores-Flores, 2021; Hatisaru, 2022; Rodríguez-Nieto et al., 2023a, 2024), but internally as they happen in the brain in the different lobes. On the other hand, it is pointed out about the definition of the mathematical connection where it has always been mentioned that it is understood as a cognitive process (García-García & Dolores-Flores, 2018; Rodríguez-Nieto et al., 2022c), but it has emerged the question: how are these cognitive processes studied?, and as an answer we have the results of this research which show the possible brain areas activated during problem-solving and their relationship with mathematical connections.

One of the important aspects of this work is that both the teacher and the student solved the corresponding part of the problem similarly, that is, they established the same mathematical connections (procedural, meaning, different representations (alternate, equivalent and from a horizontal mathematization view), part-whole and feature), for example, P1 and P2 when executing the operations $3\ cm * 4.5\ cm = 13.5\ cm^2$ and $28\ cm * 4.5\ cm = 1.036\ cm^2$, respectively from a theoretical-practical view of neuro-mathematics activate the connections in an orderly manner in the parietal, temporal lobes, prefrontal cortex and the frontal lobe that activate mathematical practice 4 evidencing the important articulation to detail the analysis of connections. In general, the proposal of a connection model is important, contributing to the functioning of the connection seen metaphorically as the tip of an iceberg (Rodríguez-Nieto et al., 2022c), see **Figure 13**.

Figure 13 shows neuro-mathematical connections linked to a mathematical operation performed by P1, highlighting that the mathematical connections depend on the neuro-mathematical connections found in the brain that were explored based on previous studies (De la Serna, 2020; Dehaene et al., 2003; Olkun, 2022), new research could develop studies applying the *electroencephalogram* technique that evaluates the electrical activity of neurons (and connections) that are in the cortex and the cerebral hemispheres considering its four lobes (frontal, temporal, parietal and occipital). This test is performed using electrodes placed on the subject's scalp and with it, the connections activated during the resolution of mathematical problems would be delved into, for example, in Verschaffel et al. (2016) this technique was used with arithmetic problems, but not emphasizing mathematical connections.

Furthermore, this research contributes to the data analysis method that emerged in research on mathematical connections from an articulated ETC-OSA view (Rodríguez-Nieto et al., 2024), considering that the new phase of identification of neuro-mathematical connections has now been incorporated, useful to analyze various phenomena in mathematics education and in other areas of knowledge

FINAL REFLECTIONS

With this type of research that articulates ETC-OSA with neuro-mathematics, the importance of establishing mathematical and neuro-mathematical connections is further corroborated, reporting the novelty of the detailed study of the connections of a teacher and his student in the resolution of a geometric problem, where it can be said that different aspects of the problem could be processed in different ways, taking into account that the left hemisphere tends to be more linked to analytical and verbal processing, while the right hemisphere could participate in spatial perception and artistic expression.

We are sure that, on the one hand, the tools reported in this network would help teachers, students, and researchers to analyze the connections in more detail and find the possible causes of the errors that students make in solving problems, also ensuring detailed study of the properties, concepts, procedures, and extra-mathematical tasks as suggested in De Gamboa et al. (2023). For example, if a student does not activate the procedural connection, it was because he possibly used the formula $v = a * b * c$ inappropriately, or he made a mistake in the execution of arithmetic operations linked to the associative property, that is, the cause of the error (disconnection) or the student's personal connection is explained.

On the other hand, it is essential to mention the limitations of this work related to the fact that only a geometric task was used where the teacher and the student agree on the procedures carried out and connections activated, but if the task changes, other types of mathematical connections could emerge and neuro-mathematics, then this work can be extended and enriched in future research.

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Ethical statement: The authors stated that the study is a theoretical integration that required the proper use of the pairs of networking of theories strategies and the review of the literature considering the proper use of APA standards and the approval of an ethics committee is not necessary. The authors further stated that the educational institution allowed the authors to carry out the study with a short episode of a class. It should be noted that one of the authors of the article was a professor in service at said institution, therefore, there are no inconveniences with this ethical aspect. To guarantee the anonymity and protect the identity of the participants, labels P1 and P2 were used and their faces were covered. Finally, the authors stated that the participants and the educational institution were informed that this article has educational purposes and not economic or political ones.

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