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Grade 11 learners' views on their engagement with mathematics through mathematical modelling

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Abstract
This study aimed at determining grade 11 learners' views on their engagement with mathematics through mathematical modelling**.** The research participants came from three selected schools in the Tshinane Circuit. These participants included three grade 11 mathematics teachers and 60 grade 11 mathematics learners. Twenty grade 11 mathematics learners were selected from each of the three selected schools, namely school A, school B, and school C. The participants were selected through purposive sampling. The student engagement in mathematics scale which is a self-report measure was used to assess three dimensions of student engagement (social, emotional, and cognitive) on their engagement with mathematics through mathematical modelling. Data analyzed descriptive analysis method using statistical package for the social sciences version 20 and interpreted in terms of theoretical framework of the study based on student engagements and mathematical modelling which is defined as using mathematics to explain and define the events in real life, to test ideas and to make estimations about real life events. When student views were analyzed, many learners expressed that they do not really enjoy solving problems with graphing linear functions. Most of them believe that they may understand graphing of linear functions better if another approach is used in teaching them. If employed regularly, the mathematical modelling approach may help to improve the learners' understanding by improving their focus and helping them remember the learning expectations in mathematics.

Keywords: social, emotional, cognitive, student's engagement, learning environment

INTRODUCTION

"Curricula reforms in many Western countries, especially at secondary level have emphasized mathematical modelling as an important element in an up-to-date mathematics secondary curriculum preparing for further education" (Blomhøj & Carreira, 2009, p. 1). The South Africa mathematics curriculum and Ethiopian education system can be seen as one curriculum among the "up-to-date mathematics secondary curricula": the curriculum and assessment policy statement (CAPS) for grade 10-grade 12 encourages the learning and teaching of mathematics through mathematics modelling (Department of Basic Education [DBE], 2011, p. 8). This is clearly stated under learning specific aims as follows:

> Mathematical modelling is an important focal point of the curriculum. Real life problems should be incorporated into all sections whenever appropriate. Examples used should be realistic and not contrived. Contextual problems should include issues relating to health, social, cultural, scientific, political and environmental issues whenever possible (DBE, 2011, p. 8).

It is uncertain to what extent SA teachers and learners have adopted this tool in mathematics instruction in a consistent and integrated way. It would be valuable to see whether there is a difference in the understanding, interest and love for mathematics among learners who modelled mathematical ideas and those who did not.

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Contribution to the literature

- Learners in the 21st century in the world need teachers support/scaffolding in the learning process because of the advancements made in digital era and so teaching methods should be revisited.
- The study employed Mathematical modeling in teaching functions and identified students engagements in learning the topic.
- The study reveals that students expressed positive engagements towards the use of Mathematical modeling for learning functions and more satisfied with the activities included during interventions.

In summary, some general notions emerged from this background which directed the argument towards the formulation of the problem statement as follows:

- Effective mathematics teaching and learning strategies generate proficient learners.
- Active engagement with mathematics results in enthusiastic and involved learners.
- Good results are not always indicative of true understanding and love for mathematics.
- Authentic mathematising includes true engagement and conceptual understanding.
- Mathematical modelling holds the potential for engagement and conceptual understanding.
- The SA curriculum makes provision for mathematical modelling at the senior level.

Connecting mathematics with the real world seems to be the main challenge in many mathematics classrooms. Frejd (2014) assert that model building should be a stimulant in mathematical learning if it is properly implemented, as is the case in the countries which are deemed to be mathematically successful.

THEORETICAL ORIENTATIONS ON MATHEMATICAL MODELLING

Different authors have defined mathematical modelling in different ways. "Mathematical modelling can be defined as using mathematics to explain and define the events in real life, to test ideas and to make estimations about real life events" (Arseven, 2015, p. 973). It is one of the main elements of a realistic approach to teaching mathematics, as it is meant to create learners who can solve real problems. The world of mathematics is not just natural, but a result of human activity, particularly from resilient intellectuals who persistently seek trends and solutions to daily situations. Mathematics will continually be discovered and perfected by such people, including classroom learners. The researcher therefore finds it necessary to have a look at realistic education and specifically how mathematical modelling as a strategy within realistic mathematics education (RME) may serve the goal of learner engagement.

RME is a type of education which was developed in the Netherlands and it follows guidelines which are intended to make mathematical learning real, hence the term "realistic". According to Sumirattana et al. (2017), the use of realistic situations can help learners in developing mathematical concepts and procedures in solving problems by linking models to their lived experience of the real world. "Although 'realistic' situations in the meaning of 'real-world' situations are important in RME, 'realistic' has an even broader connotation in modelling mathematics: It means learners are offered problem situations which they can imagine" (Van den Heuvel-Panhuizen & Drijvers, 2014, p. 521).

According to Van den Heuvel-Panhuizen and Drijvers (2014), the interpretation of "realistic" traces back to the Dutch expression "zich REALISEren", which means to imagine. This implies being able to portray something that is not always tangibly present but can happen or occur in the virtual space of the mind of the learner, through the imagination. The stem of the word "imagining" is "image"–which implies that the picture, or representation of what exists in the real world, comes into existence in the mental space of the learner as a dynamic image. This image can be manipulated, argued, analyzed and used as the basis for understanding mathematical concepts. This implies that the teacher should choose situations that are familiar to the learners or in the learners' environment for the mathematical modelling process. These situations should be imaginable to the learners, allowing them to create mental images that they can represent using pen and paper.

The word "realistic" is the one that marks the difference between RME and other forms of mathematics education, as it tries to connect mathematical symbols and models with human activity (Bedada, 2024). "Therefore, in RME problems presented to students can come from real world but also from the fantasy world of fairy tales, the formal world of mathematics, as long as problems are experimentally real in the learners' mind" (Van den Heuvel-Panhuizen & Drijvers, 2014, p. 521). Congruent with this idea, McLeod (2018) alludes that the environment in which children grow up will influence how they think and what they think about. Thus, exposure to the real situations can help to stimulate thinking among learners.

The historical development of RME resonates with the background and objectives of the present study: Van den Heuvel-Panhuizen (2000) states that in the 1960s, mathematics education in the Netherlands was dominated by a mechanist teaching approach at the formal educational level, with mathematical content derived from structural mathematics as a scientific discipline. This is to say that teachers merely demonstrated procedures on how to solve problems and the learners were expected to reproduce these procedures when solving other problems. This type of approach gave less room for the learner to reason, internalize concepts and build a reliable base for further development. According to Van den Heuvel-Panhuizen (2000), this approach triggered the development of the "new math" movement and the establishment of RME in the Netherlands.

The development of RME, according to Van den Heuvel-Panhuizen (2000), was started around 1970 by Freudenthal and his colleagues at the former Institute for the Development of Mathematics Education, which is an ancestor of the presently known Freudenthal Institute. Freudenthal's (1968) reform movement is said to have been motivated by the fact that mathematics is a human activity, and not abstract. Freudenthal (1968) states that Mathematics lessons should give learners the guided opportunity to 're-invent' mathematics by doing it. Thus, teachers do not leave learners completely to their own devices and discoveries, instead they offer learners a guided opportunity to discover mathematics by using their basic, existing knowledge and sharing their new discoveries and findings. Through guided discovery, the learners will be acting like the re-inventers of mathematics, hence equipping themselves with wide knowledge and making mathematics more memorable. Van den Heuvel-Panhuizen (2000) notes that Freudenthal (1968) termed this process mathematization.

Treffers (1987) later defined two types of mathematization–'horizontal' and 'vertical' mathematization–in mathematics education. During horizontal mathematization, learners are given real-life problems which compel them to apply the knowledge they have at hand to come up with a mathematical solution. Horizontal mathematization mainly involves the learners who try all sorts of methods and techniques, whether right or wrong, to come up with a solution. It intends to make the learners' minds active by seeking different avenues to the solution. In vertical mathematization, teachers assist learners to organize their (yet informal) findings by establishing (formal) mathematical formulae, strategies and procedures that they can apply in similar problems that they may encounter. "Thus, horizontal mathematization involves going from world of life into the world of symbols, while vertical mathematization means moving within the world of symbols" (Van den Heuvel-Panhuizen, 2000, p. 4). This implies that in vertical mathematization, the teacher gets involved to formalize the symbols, formulae and procedures. By comparing the results during horizontal mathematization and vertical mathematization, the learners should become mathematically stronger with unforgettable concepts which, to them, are similar to self-discovered concepts. According to Freudenthal (1968), these two forms of mathematization have equal significance for meaningful mathematical learning and one's chances of acquiring good mathematical skills are compromised without the other.

The inventors of RME added core principles to the teaching of mathematics to explicate RME. According to Van den Heuvel-Panhuizen and Drijvers (2014), most of these core principles were articulated originally by Treffers (1987), but were reformulated over the years, including by himself. Six distinguishable principles (Van den Heuvel-Panhuizen, 2000, p. 5) are as follows:

Activity principle: The activity principle implies that learners should be active in the learning situation rather than passive receivers of predetermined knowledge. The learners should gather their knowledge and try to confront problems which are tabled in front of them by the teacher. This principle also emphasizes the fact that mathematics is a subject that can best be learnt through practice. It should be acknowledged that most successful learners achieve through constant practice and teachers should find ways of promoting practice in learners.

Reality principle: The reality principle stresses the significance of working with real-life problems in mathematics learning, as mathematics is a product of human activity. Mathematical learning should, wherever possible, start with problems that are meaningfully attached to the learners' environment and to whatever concepts they build. It also emphasizes the importance of mathematization in problem solving. Sekerak (2010) mathematical modelling should start with contexts that need organization and before coming up with abstractions and definitions. Real situations make it easier for learners to imagine and build concepts as the memory base gets stronger, thereby helping them to create images of problems presented to them.

Level principle: The level principle emphasizes the importance of building learners' knowledge step by step. Learners should pass through different levels of understanding and progressively climb up the mathematics ladder without skipping one level. This can be done by creating models that can promote logical thinking and relate different concepts and strategies. Van den Heuvel-Panhuizen (2000) states that models serve as an important device in bridging the gap between the informal, context-related mathematics and the more informal mathematics. The models can be shifted from the learners' informal view to a particular view and finally to a general view that can be applied to many situations.

Intertwinement principle: The intertwinement principle in RME means that mathematical topics like number patterns, finance, functions, measurement, data handling and probability are interconnected and not independent from one another. To tackle a problem on finance, for instance, may require someone to gather knowledge on the other topics like number patterns and functions. The curriculum should logically be organized in such a way that the learners are pre-equipped with the necessary tools that can enable them to construct new knowledge. For instance, learners need to know the concept of measurement and types of angles first in order for them to effectively learn about Euclidean geometry. It is therefore of paramount importance for teachers to organize topics and present them logically to the learners as they climb the hierarchy of knowledge.

Interactivity principle: To make mathematics learning more effective in RME, learners need to work collectively and share their ideas and strategies. Van den Heuvel-Panhuizen (2000) states that by listening to others, finding out and discussing these findings, the learners can get ideas for improving strategies. It is the researcher's belief that there are naturally good teachers among the learners themselves and the teacher cannot exhaust all pathways in teaching a concept and such learners can assist others in the learning process. Whenever a teacher is teaching a class, the learners may not understand at the same rate or reach the expected level at the same time. The teacher may thus be forced to attend to the learners at different levels, possibly by dividing the class into smaller groups and devising learning strategies to suit the groups.

Guidance principle: Learners do not solely discover their own mathematics but are there to discover mathematical concepts that have been discovered by earlier mathematicians. It is also challenging for many learners to just rely on the textbook without guidance from the teacher. In RME, teachers have an active role in guiding the learners to progressively learn mathematics by preparing tasks that can take them step by step into the next level. The tasks should be well coordinated, with the aim of creating the desired mathematician at the end of the learning program. The teacher should also be able to provide a learning environment that can promote the construction process in the learners' memory. Van den Heuvel-Panhuizen and Drijvers (1996) state that this principle implies that in RME teachers must play an active role in designing activities that can help in shifting learners' understanding. In this way, teachers should be able to read learners' behaviors and forge a path that can enable them to reach the required level of proficiency.

As the main aim of this study was to promote mathematical modelling in the learning of linear functions in order to increase the level of learner engagement in coming to an understanding of linear relations, the researcher will explain the terms 'engagement' and 'understanding' as used in this context. The terms need clarity for the reader to understand the purpose of this research.

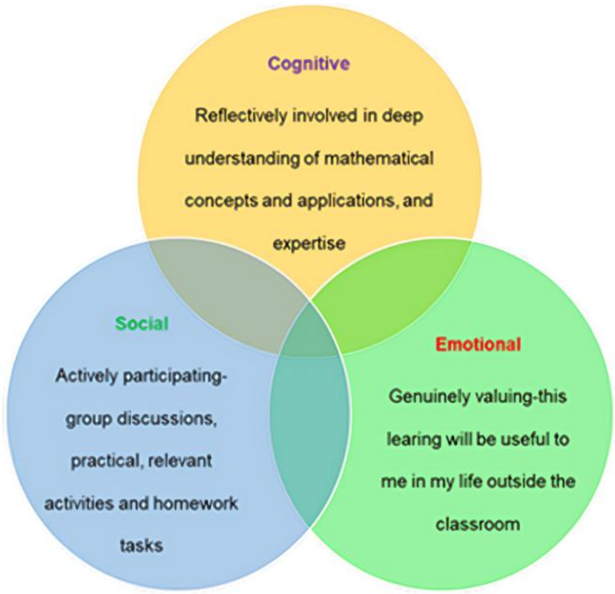

Figure 1. Framework for engagement with mathematics adapted from Fredricks et al. (2004)

Engagement

The word 'engagement' has a number of meanings, but in this study the term was in the educational context. According to the Cambridge Dictionary, engagement in the educational sense means an act of being involved with something. Thus, in this context, the term engagement implies learner involvement. "Learner engagement involves positive student behaviors, such as attendance, paying attention, and participation in class, as well as the psychological experience of identification with school and feeling that one is cared for, respected, and part of the school environment" (Anderson et al., 2004, p. 97). Knowing how to enhance learner engagement helps in encouraging positive development for the learner. It is believed that learner engagement is necessary for learners to gain knowledge and skills to succeed in their programs.

Fredricks et al. (2004) define learner engagement as a meta-construct that includes behavioral, emotional, and cognitive engagement (**Figure 1**).

According to Fredricks et al. (2004), social engagement is the term used to describe students' regular social interactions with peers that are connected to the course material. For instance, today I chatted to other kids about math, assisted other kids with arithmetic when they needed help, discussed ideas and resources with other kids in math class, and students in math class assisted one another.

According to Fredricks et al. (2004), emotional involvement refers to students' perceptions of their relationships to the material, their enthusiasm for learning, and their enjoyment of solving issues while considering the material. The degree of positive (and negative) responses to instructors, student-teacher relationships, classmates, academics, or school is the main focus of emotional involvement, according to Fredricks and McColskey (2012). The emotive responses of students in the classroom–such as curiosity, boredom, happiness, sadness, and anxiety–are being discussed here. For instance, today's math lesson was enjoyable, but I also felt bored. Nevertheless, I enjoyed thinking about math today, found math to be interesting to study, and enjoyed the sense of solving math problems.

The degree to which students demonstrate a willingness to put effort into comprehending the material, solve challenging puzzles, and control and focus their attention on the activities at hand is known as cognitive engagement, according to Fredricks et al. (2004). Cognitive engagement is defined by Fredricks and McColskey (2012) as the learner's degree of commitment to the learning process. "It involves being strategic, thoughtful, and willing to put in the necessary effort for mastering difficult skills or understanding complex ideas," according to self-regulation. This relates to adaptability in problem solving, a desire for diligence, and constructive coping mechanisms when faced with setbacks. As one of the indicators of cognitive engagement, I made every effort in math class today. For example, it was crucial to me that I grasped the material well; I tried to learn as much as I could; and I engaged in a lot of critical thinking. The measurement of learner engagement has presented many challenges. Perth and Kinross Council (2012) state that for the young person, intellectual ability and content knowledge about a particular subject are mediated by factors such as self-regulation, study skills, social and emotional skills (e.g., cooperation, respect, and resilience), mind-set and motivation. They argue that these characteristics interact with the educational experiences such as pedagogy and quality of teaching and non-instructional aspects of the educational setting.

Fredricks et al. (2004) mention learner self-report and teacher rating of learners among some of the methods of measuring learner engagement. These have been used by other teachers in the United States of America to test learner engagement. The researcher therefore suggested that the assessment of learner engagement be made through the use of a learners' questionnaire involved aspects mentioned by Perth and Kinross Council (2015) and Fredricks and McColskey (2012)–factors such as self-regulation, study skills, social and emotional skills, mind-set and motivation.

Mathematical Engagement

Success in mathematics is normally associated with a unique type of engagement, termed mathematical modelling by some researchers. This type of engagement is the most desirable and is rarely found among many learners. According to Stephen (2011), engagement in mathematics is vital to learners' success and continuing participation in mathematics. Stephen (2011) also describes a mathematically engaged class as a class where learners have little direct teaching and work on tasks individually or with their partners and self-evaluate their tasks. The teacher's role would be to attend to a few needier or disruptive learners. Coates (2007) is of the view that learning engagement in mathematics develops different aspects of learner learning experience, namely, active learning, doing difficult activities and enriching their educational experiences by formative communication with teacher.

Mocinic (2012) identified several learning strategies that motivate learners to participate and think about the content presented to them. These learning strategies include pair work, learner discussion, brainstorming, class discussion, games involving competitions and puzzles, students' debates for students' engagement in thinking to solve problems, group work involving working to solve problems, group work involving working in a team and role plays which integrate real-world situations. Mathematics as a practicing subject requires these activities which normally lead to active engagement in mathematics. The variety of mathematical concepts which are interrelated need this variety of activities for perfect understanding and meaning.

Optimal learning in mathematics needs not just the implementation of a single type of classroom activity or just individual practice. **Figure 1** shows the framework for engagement with mathematics, citing cognitive, affective, and operative features that constitute a learner's positive engagement in mathematics. It also illustrates the pedagogical practices that demonstrate a mathematically engaged classroom. Moreover, learning mathematics is enhanced through sharing knowledge with peers, besides self-practices that promote cognitive strength.

A mathematical disengagement often occurs when a learner does not see meaning in the mathematical concepts involved or the connectivity between the concepts and the previous concept taught. This may be because the learner cannot associate the concept with real life or does not see where he can apply the concept. Pursuit of challenging problems eventually ceases, and the learner will divert to other non-mathematical activities.

STATEMENT OF THE PROBLEM

The national senior certificate diagnostic report (DBE, 2011) links a lack of fundamental or basic mathematical competencies with learners' impediments in answering complex questions. Amongst others, the report says that learners faced challenges in determining the domain and range of a function, intercepts of graphs and distances between graphs. These fundamental mathematical concepts are not understood to the extent that they can be applied in the increasingly complex and abstract problem solving needed at the FET level.

The ideal of active participation, lively interest, enthusiasm, engagement and deep conceptual understanding in mathematics compels a search for ways of promoting approaches to teaching and learning mathematics which, if effectively implemented, may promote active learner participation and result in deep understanding of the fundamental concepts of mathematics. The researcher regards mathematical modelling as one such approach which is promising for engagement and understanding of the specific troublesome topic in grade 10, namely, the graphing of functions, starting with the basic concept of graphing linear functions. Some scholars forward the environmental factor is the main attributors that affect student's engagement in learning process (Watt et al., 2017). Depending on these issues the objectives of the study were articulated as follows.

OBJECTIVES OF THE STUDY

The general objective of the study is to investigate student's engagement in learning linear function through mathematical modelling.

The specific objectives of the study were:

1. To investigate student's engagement in learning linear function through mathematical modelling in three different schools
2. To examine the extent of student's engagement in learning linear function through mathematical modelling related to schools and with each other on social, cognitive and emotional engagement wing?

RESEARCH QUESTIONS

The research questions of the study were managed by the following:

1. What is the relationship between student's engagement in learning linear function through mathematical modelling in three different schools?
2. To what extent students' engagement in learning linear function through mathematical modelling are related in schools and with each other on social, cognitive, and emotional engagement wing?

METHODOLOGY AND RESEARCH DESIGN

Population and Sampling Techniques

The research participants came from three selected schools in the Tshinane Circuit. These participants included three grade 11 mathematics teachers and 60 grade 11 mathematics learners. Three grade 11 mathematics teachers were selected from the three selected schools, namely school A, school B, and school C. The Circuit has an average population of 500 grade 11 mathematics learners per year. The participants were selected through purposive sampling. Etikan et al. (2016) state that the idea behind purposive sampling is to concentrate on people with particular characteristics, who will better be able to assist with the relevant research. The grade 11 teachers were considered to be dealing with grade 11who are at an advanced stage in dealing with linear functions. In fact, linear functions had been comprehensively covered in grade 10, according to the CAPS (DBE, 2011) curriculum.

The researcher purposefully decided to work in schools A, B, and C which were of different performance levels (purposive sampling): school A is regarded as a strong with gifted learners' depending upon national exam result whereas school B and school C are regarded as average schools.

Instruments of the Study

Learners' views about modelling were obtained through questionnaires. The questionnaires consisted of close-ended questions which were based on the 5- Likert scale. "The Likert scale is applied as one of the most fundamental and frequently used psychometric tools in educational research" (Joshi et al., 2015, p. 2). It was devised to measure the attitudes of participants by selecting their level of agreement on a scale, from always = 4, usually = 3, sometimes = 2, not usually = 1, and never = 0 and analyzed depending upon the rules indicated in the study (Machaba & Bedada, 2022). The questions allowed learners to express their different views and the way they felt about modelling as a way of understanding the graphing of functions. The learner questionnaire was translated into a vernacular for the learners to understand and clearly express their feelings. The Likert scale questions also enabled us to determine the extent to which mathematical modelling can, in their view, influence learner enthusiasm, interest and involvement in the learning of graphing of functions.

This measure is designed to assess learner's engagement in mathematics after a math class. Students are asked to complete the measure immediately after a math class. The scale measures three dimensions of engagement: social, cognitive, and emotional. Sara Rimm-Kaufman and her research team at the UVA social development laboratory have used the measure with fifth grade students.

Data Analysis Method

Data analyzed with descriptive analysis method with the help of statistical package for the social sciences version 20.

Table 1. Descriptive statistics: School A learners' questionnaire responses

Items	A	U	S	NU	N	Statistics		
						M	SE	SD
Descriptive statistics school A: Cognitive engagement								
I know what the learning expectations are in mathematics.	9 (45%)	3 (15%)	3 (15%)	4 (20%)	1 (5%)	2.8000	.28654	1.28145
I have the materials that I need to achieve my expectations in mathematics.	5 (25%)	5 (25%)	8 (40%)	2 (10%)	0 (0%)	2.6500*	.22094	.98809
In class I am willing to participate.	12 (60%)	4 (20%)	2 (10%)	1 (5%)	1 (5%)	3.3000	.23056	1.03110
I am encouraged when I am rewarded for my efforts.	5 (25%)	2 (10%)	8 (40%)	4 (20%)	1 (5%)	2.3000*	.27242	1.21828
My teacher seems to motivate me as a person to work hard.	8 (40%)	8 (40%)	3 (15%)	1 (5%)	0 (0%)	3.1500	.19568	.87509
I am encouraged by others in my classroom to improve each day.	5 (25%)	3 (15%)	6 (30%)	6 (30%)	0 (0%)	2.3500*	.26433	1.18210
I make sure that I find time to study mathematics daily.	8 (40%)	4 (20%)	8 (40%)	0 (0%)	0 (0%)	3.0000	.20520	.91766
Aggregated mean						2.7929**	.22504	1.00640
Descriptive statistics school A: Social engagement								
I am willing to help my classmates to achieve.	6 (30%)	1 (5%)	6 (30%)	5 (25%)	2 (10%)	2.2000*	.31288	1.39925
I like sharing my work with my classmates.	5 (25%)	3 (15%)	7 (35%)	5 (25%)	0 (0%)	2.4000*	.25547	1.14248
I feel comfortable turning to others to help me achieve.	10 (50%)	2 (10%)	3 (15%)	4 (20%)	1 (5%)	2.8500*	.29267	1.30888
My teacher talks to me about my progress on a regular basis.	4 (20%)	6 (30%)	9 (45%)	0 (0%)	1 (5%)	2.6000*	.22243	.99472
I attend every mathematics lesson each day, wanting to learn.	11 (55%)	5 (25%)	1 (5%)	3 (15%)	0 (0%)	3.2000	.24709	1.10501
Aggregated mean						2.8167**	.19418	.86839
Descriptive statistics school A: Emotional engagement								
I enjoy solving problems on graphs of linear functions.	4 (20%)	4 (20%)	10 (50%)	1 (5%)	1 (5%)	2.4500	.23480	1.05006
I believe I can understand graphs of linear functions If I am taught in another way.	8 (40%)	7 (35%)	5 (25%)	0 (0%)	0 (0%)	3.1500	.18173	.81273
I am beginning to change my attitude towards graphs of linear function.	5 (25%)	8 (40%)	6 (30%)	1 (5%)	0 (0%)	2.8500	.19568	.87509
Aggregated mean						2.8167**	.19418	.86839

Note. A: Always; U: Usually; S: Sometimes; NU: Not usually; N: Never; M: Mean; SE: Standard error; & SD: Standard deviation

PRESENTATION OF DISCUSSION AND FINDINGS

Learner Questionnaire Responses on Mathematical Modelling

This section looks at the findings from the learners' responses from the learners' questionnaire. The aim of getting the learner responses was to assess the learners' views on mathematical modelling, especially after their exposure to the mathematical modelling approach. The learners' views could help in determining their level of engagement. The findings will be presented per school and will be analyzed in the following categories:

- Availability of resources
- Learner engagement
- Teacher role
- Learner engagement on linear functions

The responses from the three groups of learners are presented in tables. The responses of the whole group of the three schools are also summarized after the individual analysis.

Discussion

The following section shows the summary of results from the learners' questionnaire. These are presented and analyzed school by school mode with fixed instruments which has reliability of Cronbach's alpha 0.988 which was highly reliable. To answer research question 1 and research question 2 we have investigated learner's responses on each school starting from schools A, B, and C.

Table 1 shows school A learners' responses from the questionnaire (learner self-report).

Table 2 argument was granted with the following non-parametric correlations in which aggregated mean was calculated between the three themes within school

Table 2. Descriptive statistics aggregated mean engagement of the three themes in school A and Kurtosis

Themes for school A	N	Minimum	Maximum	Mean	Standard deviation		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	SE	Statistic	Statistic	SE
Cognitive engagement	20	1.00	4.00	2.7929	.22504	1.00640	-1.248	.992
Social engagement	20	1.00	4.00	2.8167	.19418	.86839	-.660	.992
Emotional engagement	20	1.00	4.00	2.8167	.19418	.86839	-.660	.992
Valid N (listwise)	20							

Note. SE: Standard error

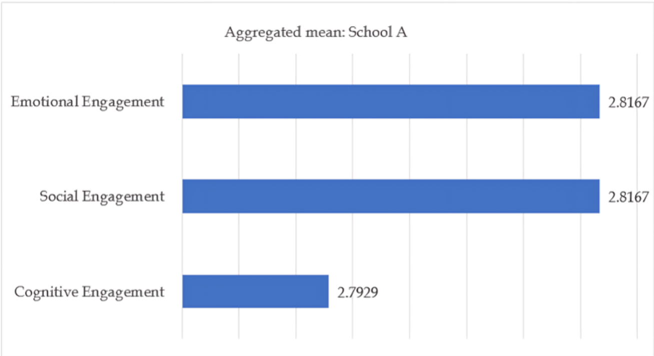

Figure 2. Aggregated means of school student's response towards each engagement-1 (Source: Authors' own elaboration)

A which was supported by the finding done by Fredricks et al. (2004) in terms intersections. We show that all three themes have positive correlation with kurtosis negative signs indicating the data are grouped in one direction in all schools. If a distribution has negative kurtosis, it is said to be platykurtic, which means that it has a flatter peak and thinner tails compared to the normal distribution. This means that there are more data values that are located near the center or mean and less data values near the mean and less data values are located on the tails. Graphically the aggregated means of school A in terms of engagement was depicted in **Figure 2**.

The mean values of students towards the willing they have to participate in the classroom learning process, teacher scaffolding to make students work hard and the time to study the course are 3.30, 3.15, and 3.00, respectively indicating that students are more engaged towards learning than other instruments in these themes. Seventy-five percent of the learners said that they always or usually or sometimes know what the learning expectations in mathematics are, while 25% of them said that they do not usually or never know the learning expectations in mathematics. Fifty percent of the learners felt that they have the materials that are needed for them to achieve in mathematics. This shows that a sizeable number of grade 11 learners in school A are not sure about the learning expectations in mathematics. Most of them also feel that they do not have adequate materials that can enable them to achieve in mathematics.

Sixty percent (mean of 3.30) of the learners expressed their willingness to participate in class by saying that they either always or usually know the expectations. Only 20% of the learners felt that they sometimes, not usually or never participate in class. Thirty-five percent of the learners said that they are always or usually encouraged by rewards for their efforts, while 40% of the learners felt that they are sometimes encouraged. The remaining 25% felt that they do not usually or never get encouraged by rewards for their efforts. Forty percent of the learners were of the opinion that they always or usually get encouraged by classmates to improve their performance, while the other 60% felt that they only sometimes or not usually get encouraged. Forty percent of the learners said that they usually or always like sharing their work with their classmates, while 35% sometimes share their work with their classmates. The above views from the learners mean that the level of interaction among learners is not so high and a number of learners may not feel free to interact. The willingness to participate shown among most learners shows that they are eager to achieve and probably there are certain factors which may hinder them from doing so. Extrinsic motivation also seems to be lacking in most learners as outside forces like rewards and encouragement from classmates have little effect.

Eighty percent (mean of 3.15) of the learners felt that their teacher usually or always motivates them as individuals to work hard, whereas 20% felt that the teacher sometimes or does not usually motivate them individually. Fifty percent of the learners answered that their teacher always or usually talks to them individually about their progress, whereas 45% were of the opinion that their teacher sometimes talks to them. Eighty percent of the learners said that they usually or always attend every mathematics lesson each day in order to learn, while the other 20% sometimes or do not usually attend every lesson each day. The teacher appears to be doing his part to attend to the learners, but some of the learners do not seem to fully cooperate.

Forty percent of the learners were of the opinion that they usually or always enjoy solving problems involving graphs of linear functions, while 50% of the learners sometimes enjoy this. Seventy-five percent of the learners thought that they could understand graphs of linear functions if other approaches are used in teaching graphing of linear functions, while the other 25% thought that sometimes they could understand better if another approach was used. Sixty-five percent of the learners felt that they are usually or are beginning to change their attitude to graphs of linear functions, while

Table 3. Non-parametric correlations of instruments: School A

Themes			Cognitive engagement	Social engagement	Emotional engagement
Kendall's tau_b	Cognitive engagement	Correlation coefficient	1.000	.945**	.945**
		Sig. (2-tailed)	-	.000	.000
		N	20	20	20
	Social engagement	Correlation coefficient	.945**	1.000	1.000**
		Sig. (2-tailed)	.000	-	-
		N	20	20	20
	Emotional engagement	Correlation coefficient	.945**	1.000**	1.000
		Sig. (2-tailed)	.000	-	-
		N	20	20	20
Spearman's rho	Cognitive engagement	Correlation coefficient	1.000	.984**	.984**
		Sig. (2-tailed)	-	.000	.000
		N	20	20	20
	Social engagement	Correlation coefficient	.984**	1.000	1.000**
		Sig. (2-tailed)	.000	-	-
		N	20	20	20
	Emotional engagement	Correlation coefficient	.984**	1.000**	1.000
		Sig. (2-tailed)	.000	-	-
		N	20	20	20

Note. **Correlation is significant at the 0.01 level (2-tailed)

30% sometimes felt that they were beginning to change their attitude towards the graphs of linear functions. Considering the above views from the learners, it may be deduced that most of the learners have some interest in graphs of linear functions. If an innovative approach like mathematical modelling can be applied for a long time or consistently, the learners may understand the concept of graphing of linear functions better, since most learners agreed that they were beginning to change their attitude towards the concept.

About 80% of the learners in school A showed increased attention to the teacher's advice and questions. The discussions among the learners and their facial expressions demonstrated curiosity among the learners for learning linear graphs of functions. This signaled that the learners' emotional engagement was generally good. Cognitively, the learners demonstrated a fairly good level of commitment by discussing actively in pairs or groups. This was a sign of fairly good cognitive engagement. Joshi et al. (2022) associate learners' motivation and self-directed learning as measures of cognitive engagement. Their enthusiasm to come up with solutions was remarkably good. They also interacted closely with their peers, with eagerness to show their individual understanding to others. They were not shy about sharing their knowledge. This was a good indicator of social engagement in mathematics.

In general, **Figure 2** indicate that students are more engaged in social and emotional than cognitive engagement in school A even though they are positively skewed please see **Table 2** and **Table 3**.

Table 4 shows school B learners' responses from the questionnaire (learner self-report). About 50% of the learners seemed to know what learning expectations are required in mathematics. Forty-five percent of the learners said that they usually or always have the materials required for achievement in mathematics, while 55% said that they sometimes have these materials. This means that a large number of learners in school B are unsure about what is expected in mathematics and the resources that are required for proper learning of mathematics.

About 90% of the learners expressed their willingness to participate in class. About half of the learners seemed not to be encouraged by rewards for their efforts. Most learners are not encouraged by their classmates to improve their performance each day and they do not like sharing their work with their colleagues. Sixty-five percent of the learners always find time to study mathematics daily, while 25% sometimes do so. Equal percentages of 30% of the learners expressed their willingness to help their classmates to achieve or not achieve, while 40% sometimes help their classmates. Sixty percent of the learners feel free to consult others to help them to achieve. The above views by school B learners show that despite their willingness to achieve they lack the energy to achieve and most of them do not collaborate with their classmates.

Most of the learners affirmed that their teacher motivates them to work hard as a person. Thirty-five percent of the learners said that their teacher does not regularly talk to them about their progress. About 80% of the learners said that they attended the mathematical lessons usually or always, while 20% of the learners sometimes attend, or do not attend the lessons. These learner responses reveal that the teacher has been doing his best to help the learners to achieve, but some of the learners appear not to cooperate by not attending the lessons.

Table 4. Descriptive statistics: School B learners' questionnaire responses

Items	A	U	S	NU	N	Statistics		
						M	SE	SD
Descriptive statistics school B: Cognitive engagement								
I know what the learning expectations are in mathematics.	9 (45%)	1 (5%)	9 (45%)	1 (5%)	0 (0%)	2.9000	.23952	1.07115
I have the materials that I need to achieve my expectations in mathematics.	4 (20%)	5 (25%)	11 (%)	0 (0%)	0 (0%)	2.6500	.18173	.81273
In class I am willing to participate.	18 (90%)	2 (10%)	0 (0%)	0 (0%)	0 (0%)	3.9000*	.06882	.30779
I am encouraged when I am rewarded for my efforts.	7 (35%)	1 (5%)	4 (20%)	6 (30%)	2 (10%)	2.2500	.33146	1.48235
My teacher seems to motivate me as a person to work hard.	9 (45%)	6 (30%)	4 (20%)	1 (5%)	0 (0%)	3.1500	.20869	.93330
I am encouraged by others in my classroom to improve each day.	5 (25%)	0 (0%)	4 (20%)	8 (40%)	3 (15%)	1.8000	.32118	1.43637
I make sure that I find time to study mathematics daily.	13 (65%)	1 (5%)	5 (25%)	1 (5%)	0 (0%)	3.3000	.23056	1.03110
Aggregated mean						2.8500	.20532	.91822
Descriptive statistics school B: Social engagement								
I am willing to help my classmates to achieve.	3 (15%)	3 (15%)	8 (40%)	4 (20%)	2 (10%)	2.0500	.26631	1.19097
I like sharing my work with my classmates.	2 (10%)	2 (10%)	9 (45%)	5 (25%)	2 (10%)	1.8500	.24360	1.08942
I feel comfortable turning to others to help me achieve.	12 (60%)	0 (0%)	6 (30%)	1 (5%)	1 (5%)	3.0500	.28539	1.27630
My teacher talks to me about my progress on a regular basis.	8 (40%)	1 (5%)	4 (20%)	5 (25%)	2 (10%)	2.4000	.33561	1.50088
I attend every mathematics lesson each day, wanting to learn.	15 (75%)	1 (5%)	2 (10%)	1 (5%)	1 (5%)	3.4000	.26557	1.18766
Aggregated mean							2.550**	.25706
Descriptive statistics school B: Emotional engagement								
I enjoy solving problems on graphs of linear functions.	9 (45%)	1 (5%)	9 (45%)	1 (5%)	0 (0%)	2.9000	.23952	1.07115
I believe I can understand graphs of linear functions If I am taught in another way.	11 (55%)	4 (20%)	4 (20%)	1 (5%)	0 (0%)	3.2500	.21613	.96655
I am beginning to change my attitude towards graphs of linear function.	7 (35%)	2 (10%)	8 (40%)	2 (10%)	1 (5%)	2.6000	.27530	1.23117
Aggregated mean						2.9167**	.23305	1.04224

Note. A: Always; U: Usually; S: Sometimes; NU: Not usually; N: Never; M: Mean; SE: Standard error; & SD: Standard deviation

Fifty percent of the learners said that they usually or always enjoy solving problems on linear functions, while the rest sometimes or rarely enjoy solving problems on linear functions. Seventy-five percent of the learners believe that they could understand graphing of linear functions if it is taught in another way. Forty-five percent felt that they were beginning to change their attitude towards the graphing of linear functions.

The above responses show that about half of the learners do not seem to really enjoy graphing of linear functions. Considering these responses, the learners may need a more enticing approach to the teaching of graphs of linear functions. The mathematical modelling approach may have a positive effect if it is applied over a long period of time since 45% felt that they are beginning to change their attitude towards graphing linear functions.

The following observations were noted regarding emotional, social and cognitive engagement in school B. The learners in school B paid more attention to the teacher's instructions, though with some signs of anxiety. Some learners did their tasks with some signs of confidence, despite interpreting the aspects of linear functions wrongly. This indicates that the learners' emotional engagement was averagely good. Joshi et al. (2022) take interest and feelings, increasing happiness, comfort in practicing and reduced anxiety as measures of emotional engagement. Their level of commitment was comparably lower than that of school A with some learners spending about 8 minutes desperately failing to come up with a way forward. This was a sign of lower cognitive engagement as compared to school A. About 60% of them were free to share their ideas with their peers. Some were, however, keen to seek solutions from their peers as they encountered challenges or to compare their solutions with others. Some moved up and down the classroom to seek advice from other groups. These actions demonstrated an average level of social engagement in mathematics.

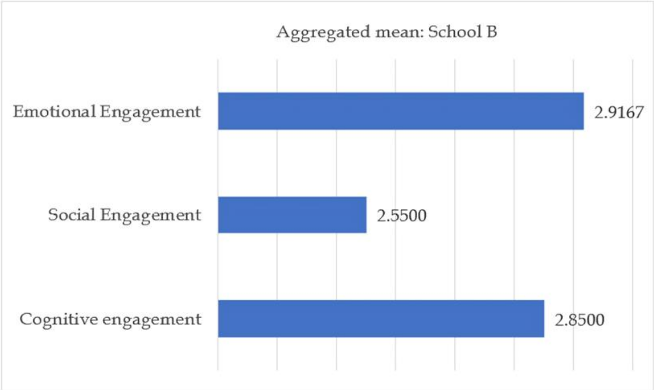

Figure 3. Aggregated means of school student's response towards each engagement-2 (Source: Authors' own elaboration)

In general, **Figure 3** indicate that students are more engaged in emotional then social than cognitive engagement in school B even though they are positively skewed please see **Table 5** and **Table 6**.

Table 7 shows school C learners' responses from the questionnaire (learner self-report).

Fifty-five percent of the learners in school C felt that they always or usually know the learning expectations in mathematics, while 40% sometimes know the learning expectations. Fifty percent felt that they have the materials that they need to achieve their expectations in mathematics, while the others felt that they either sometimes or rarely have the necessary materials. These responses show that many of the learners are unsure about the learning expectations in mathematics. Half of learners also seem not to have the materials required for them to achieve the expectations in mathematics despite the government's efforts to provide enough and relevant resources timeously to public schools.

Ninety-five percent of the learners expressed their willingness to participate in class, while only 5% felt that he or she sometimes is willing to participate. Forty-five percent of the learners showed that they are encouraged by rewards for their efforts, while the others said that they are sometimes or not encouraged by rewards. Forty-five percent of the learners said that they are motivated by classmates to improve their performance each day. Only 20% of the learners said that they always share their work with their classmates. Forty percent said that they sometimes share their work with others, while 35% rarely share their work with others. Fifty percent of the learners said that they always or usually find time to study mathematics, while 45% sometimes find time to study mathematics. Fifty-five percent of the learners expressed their willingness to help others, while 45% either sometimes or rarely help others. Eighty-five percent of the learners said that they usually or always feel comfortable to talk to others to help them to achieve. The above responses show that the learners are willing to participate in class. However, self-study seems to be lacking among about half the learners. Moreover, the learners appear not to feel free to collaborate in order to gain knowledge from colleagues.

Table 5. Descriptive statistics aggregated mean engagement of the three themes in school B and Kurtosis

Themes for school B	N	Minimum	Maximum	Mean	Standard deviation		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	SE	Statistic	Statistic	SE
Cognitive engagement	20	1.14	4.00	2.8500	.20532	.91822	-1.274	.992
Social engagement	20	.00	4.00	2.5500	.25706	1.14960	-.134	.992
Emotional engagement	20	.67	4.00	2.9167	.23305	1.04224	-.927	.992
Valid N (listwise)	20							

Note. SE: Standard error

Table 6. Non-parametric correlations of instruments: School B

Themes		Cognitive engagement	Social engagement	Emotional engagement
Cognitive engagement	Pearson correlation	1	.960**	.981**
	Sig. (2-tailed)	-	.000	.000
	Sum of squares and cross-products	16.019	19.250	17.845
	Covariance	.843	1.013	.939
	N	20	20	20
Social engagement	Pearson correlation	.960**	1	.942**
	Sig. (2-tailed)	.000	-	.000
	Sum of squares and cross-products	19.250	25.110	21.450
	Covariance	1.013	1.322	1.129
	N	20	20	20
Emotional engagement	Pearson correlation	.981**	.942**	1
	Sig. (2-tailed)	.000	.000	-
	Sum of squares and cross-products	17.845	21.450	20.639
	Covariance	.939	1.129	1.086
	N	20	20	20

Note. **Correlation is significant at the 0.01 level (2-tailed)

Table 7. Descriptive statistics: School C learners' questionnaire responses

Items	A	U	S	NU	N	Statistics		
						M	SE	SD
Descriptive statistics school C: Cognitive engagement								
I know what the learning expectations are in mathematics.	10 (50%)	1 (5%)	8 (40%)	1 (5%)	0 (0%)	3.0000	.24061	1.07606
I have the materials that I need to achieve my expectations in mathematics.	9 (45%)	1 (5%)	6 (30%)	3 (15%)	1 (5%)	2.7000*	.30000	1.34164
In class I am willing to participate.	17 (85%)	2 (10%)	1 (5%)	0 (0%)	0 (0%)	3.7500	.16018	.71635
I am encouraged when I am rewarded for my efforts.	8 (40%)	1 (5%)	4 (20%)	3 (15%)	4 (20%)	2.3000*	.36346	1.62546
My teacher seems to motivate me as a person to work hard.	9 (45%)	4 (20%)	5 (25%)	0 (0%)	2 (10%)	2.9000	.28928	1.29371
I am encouraged by others in my classroom to improve each day.	8 (40%)	1 (5%)	7 (35%)	1 (%)	3(%)	2.5000	.32847	1.46898
I make sure that I find time to study mathematics daily.	8 (40%)	2 (10%)	9 (45%)	1 (5%)	0(0%)	2.8000	.25752	1.15166
Aggregated mean						2.8500*	.26085	1.16655
Descriptive statistics school C: Social engagement								
I am willing to help my classmates to achieve.	9 (45%)	2 (10%)	4 (20%)	2 (10%)	3 (15%)	3.9500	.05000	.22361
I like sharing my work with my classmates.	5 (25%)	0 (0%)	8 (40%)	6 (30%)	1 (5%)	2.1000	.28004	1.25237
I feel comfortable turning to others to help me achieve.	16 (80%)	1 (5%)	2 (10%)	1 (5%)	0 (0%)	3.6000	.19735	.88258
My teacher talks to me about my progress on a regular basis.	2 (10%)	3 (15%)	11 (55%)	3 (15%)	1 (5%)	2.1000*	.21643	.96791
I attend every mathematics lesson each day, wanting to learn.	19 (95%)	1 (5%)	0 (0%)	0 (0%)	0 (0%)	3.9500	.05000	.22361
Aggregated mean						3.1400	.13849	.61934
Descriptive statistics school C: Emotional engagement								
I enjoy solving problems on graphs of linear functions.	11 (55%)	3 (15%)	5 (15%)	1 (5%)	0 (0%)	3.2000	.22478	1.00525
I believe I can understand graphs of linear functions If I am taught in another way.	13 (65%)	2 (10%)	5 (25%)	0 (0%)	0 (0%)	3.4000	.19735	.88258
I am beginning to change my attitude towards graphs of linear function.	12 (60%)	4 (20%)	4 (20%)	0 (0%)	0 (0%)	3.4000	.18353	.82078
Aggregated mean						2.8167*	.19418	.86839

Note. A: Always; U: Usually; S: Sometimes; NU: Not usually; N: Never; M: Mean; SE: Standard error; & SD: Standard deviation

Sixty-five percent of the learners said that their teacher usually or always motivates them to work hard as a person. On the other hand, 25% were of the opinion that their teacher sometimes motivates them, while 10% believed that the teacher never motivates them. Twenty-five percent of the learners said that their teacher talks to them about their progress on a regular basis, while 55% of the learners said that their teacher sometimes talks to them about their progress and 20% felt that their teacher rarely or never talks to them. All the learners asserted that they attend every mathematics lesson each day in order to learn mathematics. These views from the learners show that the teacher is playing an active role in motivating the learners though there may be a few learners who may be neglected. The teacher appears to be trying his best in providing planned lessons as the learners felt that they attended every lesson. However, the learners do not appear to support the teacher in his efforts to uplift them in mathematics.

Seventy percent of the learners were of the view that they always or usually enjoy solving problems on graphing of functions, while 20% felt that they sometimes enjoy solving the problems. Seventy-five percent of the learners felt that they could understand graphs of linear functions if it was taught using another approach, whereas 25% of the learners were not so sure if they would understand better if another approach was employed. Eighty percent of the learners felt that they are beginning to change their attitude towards the graphing of linear functions, while 20% thought that they have some feeling of change of attitude toward the graphing of linear functions. With these views it may be established that the majority of these learners have a positive attitude towards graphing linear functions. The learners' attitudes may improve if more appealing approaches can be employed as the learners feel that they may understand better if a different approach is used. Perhaps the mathematical modelling approach could improve their attitude if it is employed for a long

Table 8. Descriptive statistics aggregated mean engagement of the three themes in school C and Kurtosis

Themes for school B	N	Minimum	Maximum	Mean	Standard deviation		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	SE	Statistic	Statistic	SE
Cognitive engagement	20	.29	4.00	2.8500	.26085	1.16655	-.709	.992
Social engagement	20	1.40	4.00	3.1400	.13849	.61934	2.115	.992
Emotional engagement	20	1.00	4.00	2.8167	.19418	.86839	-.660	.992
Valid N (listwise)	20							

Note. SE: Standard error

time as most learners felt that their attitude is beginning to change.

Regarding emotional, cognitive and social engagement, the following observations were made. The learners from school C paid added attention to their teacher's advice, but when it came to writing their tasks, their happiness was generally lower. It appeared as if most of the learners did not see the essence of studying graphs of linear functions. This was a clear sign of low emotional engagement in mathematics. They were not so active in finding solutions to the given problems with some getting docile. It was a bit challenging for them to interpret the problems and convert them to mathematical language. This was another sign of lower cognitive engagement in mathematics among most learners. Most of the learners were stuck to their seats with limited movement and discussing with their immediate neighbors. About 15% of them could be seen moving up and down the classroom to seek advice from their peers. Some of the discussions among about 10% of these learners usually degenerated from mathematical discussions to social discussions involving other social issues. This was not a good sign of good social engagement in mathematics, Joshi et al. (2022) cite formation of study groups and participation in the groups as some of the measures of social engagement in mathematics.

Additional analysis was conducted to show the strength correlation of the three-theme indicated in **Table 8**.

In general, **Figure 4** indicates that students are more engaged in social and cognitive than emotional engagement in school C even though they are positively skewed please see **Table 8** and **Table 9**.

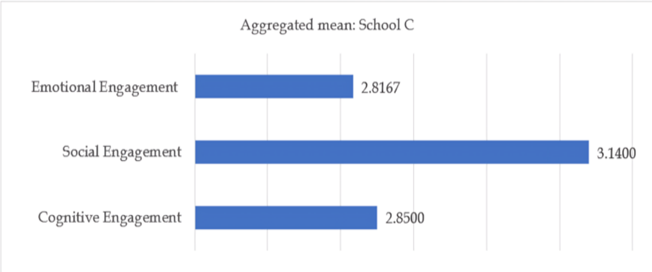

Figure 4. Aggregated means of school student's response towards each engagement-3 (Source: Authors' own elaboration)

Comparison Between Three Schools Towards Engagement Towards Learning Mathematics

Figure 5 indicated that the mean comparison of student's engagement in each school. **Figure 5** also indicates that about 34.4 % ,33% and 32.6% of student's show engagement in their learning in schools C, A, and B, respectively.

PRESENTATION OF CONCLUSIONS AND IMPLICATION OF FUTURE STUDY

Engagement is known to be associated with positive school outcomes and influenced by environmental factors (Watt et al., 2017). The difference in student's engagement into the three themes is because of environmental factors need to be investigated for future study. For instance, the learners from three schools (A, B, and C) are not sure of the learning expectations in mathematics and this may make them lose concentration or focus. To have motivation to study, a learner must know what the assessment objectives are and where the concepts taught may be applied in life. The learners also

Table 9. Non-parametric correlations of instruments: School C

Themes		Cognitive engagement	Social engagement	Emotional engagement
Cognitive engagement	Pearson correlation	1	.884**	.946**
	Sig. (2-tailed)	-	.000	.000
	N	20	20	20
Social engagement	Pearson correlation	.884**	1	.931**
	Sig. (2-tailed)	.000	-	.000
	N	20	20	20
Emotional engagement	Pearson correlation	.946**	.931**	1
	Sig. (2-tailed)	.000	.000	-
	N	20	20	20

Note. **Correlation is significant at the 0.01 level (2-tailed)

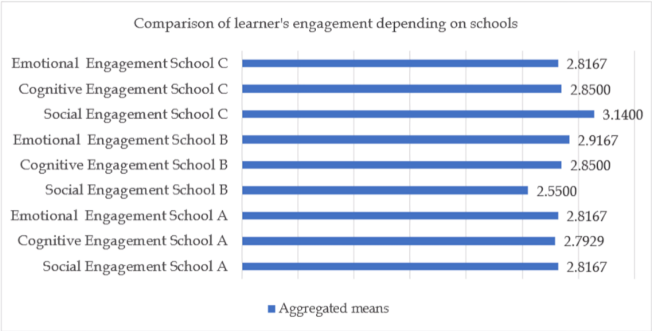

Figure 5. Aggregated means of all schools student's response towards each engagement (Source: Authors' own elaboration)

thought that they do not have the materials to help them to achieve the expectations in mathematics. It remains to be investigated what the missing materials required by the learners. Most of the learners from the three schools showed that they are willing to participate in class and other activities given to them by the scaffolders. In school the scaffolders do not motivate students compared to the other two schools B and C. The learners do not feel free to collaborate with their colleagues and this may be a big setback to their learning.

Self-motivation, which is key for any learner, also seems to be missing in the learners. The learner should be self-directed in his studies in this South African democratic society where the teacher is not allowed to compel the learner to studying. The learners from all the schools involved agreed that teachers are trying their best in their duty to guide them. The learners appear to be the ones who are not doing their duty to study the subject and not just rely on classroom activities. Most teachers may be overburdened by constantly monitoring lazy learners and having to re-teach them the concepts instead of moving forward with the syllabus.

Many learners expressed that they do not really enjoy solving problems on graphing linear functions (Bedada, 2024). Most of them believe that they may understand graphing of linear functions better if another approach is used in teaching them. If employed regularly, the mathematical modelling approach may help to improve the learners' understanding by improving their focus and helping them remember the learning expectations in mathematics. The learners felt that they were beginning to change their attitudes towards the graphing of linear functions and perhaps the mathematical modelling approach had an effect on this. Finally, the study forwards the relationship between the three themes by developing the following **Figure 6** indicating the three themes are interconnected and share the common region known as Environmental factors (can be school or teachers).

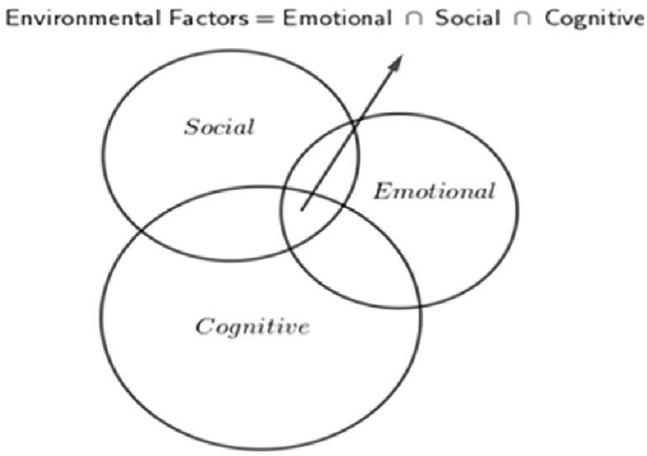

Figure 6. The region of intersection of the three Engagement themes (Source: Authors' own elaboration)

Author contributions: TB: conceptualization, software, formal analysis; **MFM:** conceptualization, methodology, writing – review & editing, supervision; **GK:** formal analysis, writing – original draft, writing – review & editing. All authors agreed with the results and conclusions.

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Ethical statement: The authors stated that the study was conducted in accordance with ethical research standards involving human participants in learning process. The study was approved by the instituitonal ethics committee of UNISA College of Education on 2021/03/10 to 2024/03/10 (Approval code: 2021/03/10/46997318/09/AM). Prior to participation, all student participants were provided with a clear explanation of the study's purpose, procedures, potential risks, and benefits. The authors further stated that consent was obtained orally, in line with approved ethical guidelines, particularly in situations where written consent was impractical or unnecessary. Students were informed that participation was entirely voluntary, that they could withdraw at any time without penalty, and that their responses would be kept confidential and anonymized.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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